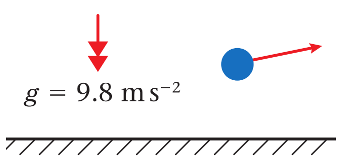
1 Mathematical models in mechanics

Mechanics is the branch of mathematics which deals with the action of forces on objects. Mechanics can be used to answer questions about many familiar situations – the motion of cars, the speed of a parachutist, the stresses in a bridge or the motion of the Earth around the Sun.

There are many factors that can complicate real-world problems. The motion of a cricket ball might be affected by the spin of the ball, the effects of air-resistance or the roughness of the pitch. We can simplify a problem by creating a **mathematical model**. This model will involve making a number of **modelling assumptions**, such as ignoring air-resistance, or treating a three-dimensional object as a particle.

Modelling assumptions

- ignore air resistance
- perfect bounce on a flat pitch
- constant force due to gravity
- model ball as a particle



Modelling assumptions can simplify a problem, and allow us to carry out an analysis of a real-life situation using known mathematical techniques. Having fewer modelling assumptions will usually make a problem more difficult mathematically.

By modelling the motion of this cricket ball, we can use mathematical techniques to predict whether it will hit the stumps.

1.1 You need to understand the significance of different modelling assumptions, and how they affect the calculations in a particular problem.

Here are some common models and modelling assumptions that you need to know.

- **Particle** – an object which is small in comparison with other sizes or lengths can be modelled as a **particle**. This means that the mass of the object can be considered to be concentrated at a single point (a particle is often referred to as a point-mass). The fact that a particle has no dimensions means that we can ignore the rotational effect of any forces that are acting on it as well as any effects due to air resistance.

- **Rod** – an object with one dimension small in comparison with another (such as a metre ruler or a beam) can be modelled as a **rod**. This means that the mass of the object can be considered to be distributed along a straight line. A rod has no thickness (it is one-dimensional) and is rigid (it does not bend or buckle).

- **Lamina** – An object with one dimension (its thickness) very small in comparison with the other two (its length and width) can be modelled as a **lamina**. This means that the mass of the object can be considered to be distributed across a flat surface. A lamina has no thickness (it is two-dimensional). For example, a sheet of paper or metal could be modelled as a lamina.

- **Uniform body** – If an object is uniform then its mass is evenly distributed over its entire volume. This means that the mass of the body can be considered to be concentrated at a single point (known as the **centre of mass**), at the 'geometrical centre' of the body. For example, an unsharpened pencil could be modelled as a uniform rod. However, once it is sharpened then its centre of mass would not be at its mid-point and we would model it as a non-uniform rod.

- **Light object** – If the mass of an object is very small in comparison with the masses of other objects, we can model it as being **light**. This means that we can ignore its mass altogether and treat it as having zero mass. Strings and pulleys are often modelled as being light.

- **Inextensible string** – If a string does not stretch under a load it is **inextensible** or **inelastic**. In M1 you will model all strings as being inextensible.

- **Smooth surface** – If we want to ignore the effects of friction, we can model a surface as being **smooth**. This means that we assume there is no friction between the surface and any object which is moving or tending to move along it.

- **Rough surface** – If a surface is not smooth it is said to be **rough**. We need to consider the friction between the surface and an object moving or tending to move along it. For example, a ski slope might be modelled as a smooth or a rough surface depending on the problem to be solved.

- **Wire** – A rigid thin length of metal, which is treated as being one-dimensional, is referred to as a **wire**. A wire can be smooth or rough. We often consider beads which are threaded on a wire.

- **Smooth and light pulley** – In M1 you will model all pulleys as being **smooth** (there is no friction at the bearing of the pulley or between the pulley and the string) and **light** (the pulley has zero mass).

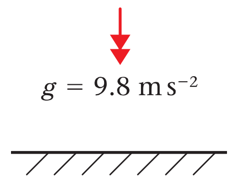
- **Bead** – A particle which can be threaded onto, and move freely along, a wire or string is called a **bead**.

- **Peg** – A support from which an object can be suspended or on which an object can rest is called a **peg**. A peg is treated as being dimensionless (it is treated as a point) and is usually fixed. A peg can be rough or smooth.

- **Air resistance** – When an object moves through the air it experiences a resistance due to friction. In M1 you will model air resistance as being negligible.

- **Wind** – Unless it is specifically mentioned, you can usually ignore any effects due to the wind in your models.

- **Gravity** – The force of attraction between all objects with mass is called **gravity**. Because the mass of the Earth is large, we can usually assume that all objects are attracted towards the Earth (ignoring any force of attraction between the objects themselves). We usually model the force of the Earth's gravity as uniform, and acting vertically downwards. The acceleration due to gravity is denoted by g and is always assumed to be constant at $9.8\,\text{m}\,\text{s}^{-2}$. This value is given on the front of the exam paper.



2 Kinematics of a particle moving in a straight line

After completing this chapter you should be able to

solve problems involving motion in a straight line with constant acceleration

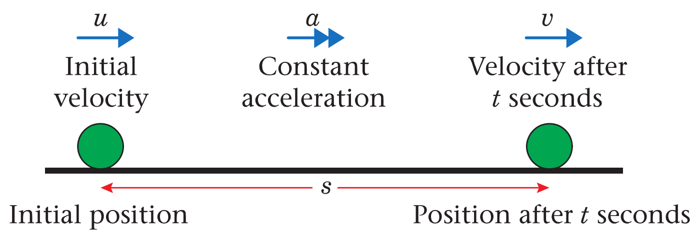
model an object moving vertically under gravity

understand distance–time graphs and speed–time graphs.

2.1 You can use the formulae $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$ for a particle moving in a straight line with constant acceleration.

■ **You need to learn this list of symbols and what they represent.**

s	displacement (distance)
u	starting (initial) velocity
v	final velocity
a	acceleration
t	time



Displacements, velocities and accelerations have directions as well as sizes (or magnitudes).

You can derive the formulae for motion in a straight line with constant acceleration.

- acceleration $= \frac{\text{change in velocity}}{\text{change in time}}$

  $a = \frac{v-u}{t}$

  $\boldsymbol{v = u + at}$

- distance moved = average speed × time

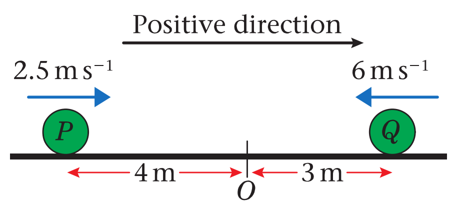
  average speed $= \frac{u+v}{2}$

  $\boldsymbol{s = \left(\frac{u+v}{2}\right)t}$

For an object moving horizontally, the positive direction is usually taken as left to right. The starting point of an object is usually taken as the origin from which displacements are measured.

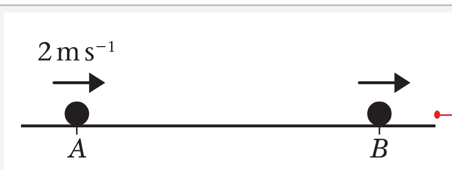
For P: $s = -4$ m and $v = 2.5\,\text{m s}^{-1}$

For Q: $s = 3$ m and $v = -6\,\text{m s}^{-1}$



Example 1

A particle is moving in a straight line from A to B with constant acceleration $3\,\text{m s}^{-2}$. Its speed at A is $2\,\text{m s}^{-1}$ and it takes 8 seconds to move from A to B. Find **a** the speed of the particle at B, **b** the distance from A to B.



Start by drawing a diagram.

$a = 3$, $u = 2$, $t = 8$, $v = ?$, $s = ?$

Write down the values you know and the values you need to find.

a $v = u + at$

$= 2 + (3 \times 8)$

$= 26$

The speed of the particle at B is $26\,\text{m s}^{-1}$.

You need v and you know u, a and t so you can use $v = u + at$.

b $s = \left(\frac{u + v}{2}\right)t$

$= \left(\frac{2 + 26}{2}\right) \times 8$

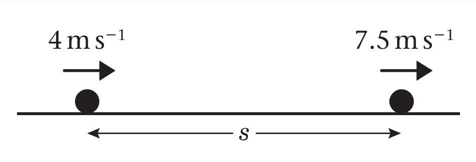
$= 112$

The distance from A to B is 112 m.

Choose the right formula then substitute in the values you know.

Example 2

A cyclist is travelling along a straight road. She accelerates at a constant rate from a speed of $4\,\text{m}\,\text{s}^{-1}$ to a speed of $7.5\,\text{m}\,\text{s}^{-1}$ in 40 seconds. Find **a** the distance she travels in these 40 seconds, **b** her acceleration in these 40 seconds.



Model the cyclist as a particle.

$u = 4,\ v = 7.5,\ t = 40,\ s = ?,\ a = ?$

a $s = \left(\frac{u + v}{2}\right)t$

$= \left(\frac{4 + 7.5}{2}\right) \times 40$

$= 230$

The distance the cyclist travels is 230 m.

b $v = u + at$

You need a and you know v, u and t so you can use $v = u + at$.

$7.5 = 4 + 40a$

Substitute the values you know into the formula. You can solve this equation to find a.

$a = \frac{7.5 - 4}{40} = 0.0875$

You could rearrange the formula before you substitute the values:

$a = \frac{v - u}{t}$

The acceleration of the cyclist is $0.0875\,\text{m}\,\text{s}^{-2}$.

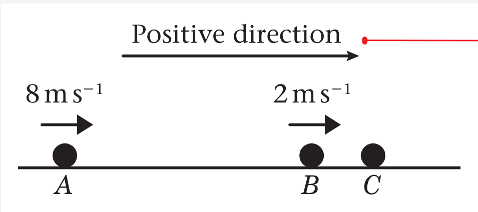
In real-life situations values for the acceleration are often quite small. Large accelerations feel unpleasant and may be dangerous.

- **If a particle is slowing down it has a negative acceleration. This is called deceleration or retardation.**

Example 3

A particle moves in a straight line from a point A to a point B with constant deceleration $1.5\,\text{m}\,\text{s}^{-2}$. The speed of the particle at A is $8\,\text{m}\,\text{s}^{-1}$ and the speed of the particle at B is $2\,\text{m}\,\text{s}^{-1}$. Find **a** the time taken for the particle to move from A to B , **b** the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration $1.5\,\text{m}\,\text{s}^{-2}$. The particle is at the point C 6 seconds after passing through the point A. Find **c** the velocity of the particle at C, **d** the distance from A to C.



Mark the positive direction on your diagram.

$u = 8, v = 2, a = -1.5, t = ?, s = ?$

The particle is decelerating, so the value of a is negative.

a $v = u + at$

$2 = 8 - 1.5t$

$1.5t = 8 - 2$

$t = \dfrac{8-2}{1.5} = 4$

The time taken to move from A to B is 4 s.

b $s = \left(\dfrac{u+v}{2}\right)t$

$= \left(\dfrac{8+2}{2}\right) \times 4 = 20$

You can use your answer from part **a** as the value of t.

The distance from A to B is 20 m.

c $u = 8, a = -1.5, t = 6, v = ?$

$v = u + at$

$= 8 + (-1.5) \times 6$

$= 8 - 9 = -1$

The velocity at C is negative. This means that the particle is moving from right to left.

The velocity of the particle is $1\,\text{m}\,\text{s}^{-1}$ in the direction $\overrightarrow{BA}$.

Remember that to specify a velocity it is necessary to give speed and direction.

d $s = \left(\dfrac{u+v}{2}\right)t$

$= \left(\dfrac{8+(-1)}{2}\right) \times 6$

Make sure you use the correct sign when substituting a negative value into a formula.

The distance from A to C is 21 m.

- **Convert all your measurements into base SI units before substituting values into the formulae.**

Measurement	SI unit
time (t)	seconds (s)
displacement (s)	metres (m)
velocity (v or u)	metres per second (m s^{-1})
acceleration (a)	metres per second per second (m s^{-2})

Example 4

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 second later the car passes a speed-trap where it is registered as travelling at $45\,\text{km h}^{-1}$. Find **a** the acceleration of the car, **b** the distance between the traffic lights and the speed-trap.

$45\,\text{km h}^{-1} = 45 \times 1000/3600\,\text{m s}^{-1}$

$= 12.5\,\text{m s}^{-1}$

Convert into SI units, using:
1 km = 1000 m
1 hour = 60 × 60 s = 3600 s

Model the car as a particle and draw a diagram.

$u = 0,\ v = 12.5,\ t = 30,\ a = ?,\ s = ?$

The car starts from rest, so the initial velocity is zero.

a $v = u + at$

$12.5 = 0 + 30a$

$a = \frac{12.5}{30} = \frac{5}{12}$

The acceleration of the car is $\frac{5}{12}\,\text{m s}^{-2}$.

This is an exact answer. If you want to give an answer using decimals, you should round to three significant figures.

b $s = \left(\frac{u+v}{2}\right)t$

$= \left(\frac{0 + 12.5}{2}\right) \times 30$

$= 187.5$

The distance between the traffic lights and the speed trap is 187.5 m.

Exercise 2A

1 A particle is moving in a straight line with constant acceleration $3\ \text{m s}^{-2}$. At time $t = 0$, the speed of the particle is $2\ \text{m s}^{-1}$. Find the speed of the particle at time $t = 6\ \text{s}$.

2 A particle is moving in a straight line with constant acceleration. The particle passes a point with speed $1.2\ \text{m s}^{-1}$. Four seconds later the particle has speed $7.6\ \text{m s}^{-1}$. Find the acceleration of the particle.

3 A car is approaching traffic lights. The car is travelling with speed $10\ \text{m s}^{-1}$. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

4 A particle moves in a straight line from a point A to point B with constant acceleration. The particle passes A with speed $2.4\ \text{m s}^{-1}$. The particle passes B with speed $8\ \text{m s}^{-1}$, five seconds after it passed A. Find the distance between A and B.

5 A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has speed $28\ \text{m s}^{-1}$. Find the speed of the car at the first signpost.

6 A particle is moving along a straight line with constant deceleration. The points X and Y are on the line and $XY = 120\ \text{m}$. At time $t = 0$, the particle passes X and is moving towards Y with speed $18\ \text{m s}^{-1}$. At time $t = 10\ \text{s}$, the particle is at Y. Find the velocity of the particle at time $t = 10\ \text{s}$.

7 A cyclist is moving along a straight road from A to B with constant acceleration $0.5\ \text{m s}^{-2}$. Her speed at A is $3\ \text{m s}^{-1}$ and it takes her 12 seconds to cycle from A to B. Find **a** her speed at B, **b** the distance from A to B.

8 A particle is moving along a straight line with constant acceleration from a point A to a point B, where $AB = 24\ \text{m}$. The particle takes 6 s to move from A to B and the speed of the particle at B is $5\ \text{m s}^{-1}$. Find **a** the speed of the particle at A, **b** the acceleration of the particle.

9 A particle moves in a straight line from a point A to a point B with constant deceleration $1.2\ \text{m s}^{-2}$. The particle takes 6 s to move from A to B. The speed of the particle at B is $2\ \text{m s}^{-1}$ and the direction of motion of the particle has not changed. Find **a** the speed of the particle at A, **b** the distance from A to B.

10 A train, travelling on a straight track, is slowing down with constant deceleration $0.6\ \text{m s}^{-2}$. The train passes one signal with speed $72\ \text{km h}^{-1}$ and a second signal 25 s later. Find **a** the speed, in km h^{-1}, of the train as it passes the second signal, **b** the distance between the signals.

11 A particle moves in a straight line from a point A to a point B with a constant deceleration of $4\ \text{m s}^{-2}$. At A the particle has speed $32\ \text{m s}^{-1}$ and the particle comes to rest at B. Find **a** the time taken for the particle to travel from A to B, **b** the distance between A and B.

12 A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a speed of $16\,\text{m s}^{-1}$ and, after moving up the hill for 40 s, he comes to rest. Find **a** the deceleration of the skier, **b** the distance from the bottom of the hill to the point where the skier comes to rest.

13 A particle is moving in a straight line with constant acceleration. The points A, B and C lie on this line. The particle moves from A through B to C. The speed of the particle at A is $2\,\text{m s}^{-1}$ and the speed of the particle at B is $7\,\text{m s}^{-1}$. The particle takes 20 s to move from A to B.

a Find the acceleration of the particle.

The speed of the particle is C is $11\,\text{m s}^{-1}$. Find

b the time taken for the particle to move from B to C,

c the distance between A and C.

14 A particle moves in a straight line from A to B with constant acceleration $1.5\,\text{m s}^{-2}$. It then moves, along the same straight line, from B to C with a different acceleration. The speed of the particle at A is $1\,\text{m s}^{-1}$ and the speed of the particle at C is $43\,\text{m s}^{-1}$. The particle takes 12 s to move from A to B and 10 s to move from B to C. Find

a the speed of the particle at B,

b the acceleration of the particle as it moves from B to C,

c the distance from A to C.

15 A cyclist travels with constant acceleration $x\,\text{m s}^{-2}$, in a straight line, from rest to $5\,\text{m s}^{-1}$ in 20 s. She then decelerates from $5\,\text{m s}^{-1}$ to rest with constant deceleration $\frac{1}{2}x\,\text{m s}^{-2}$. Find **a** the value of x, **b** the total distance she travelled.

16 A particle is moving with constant acceleration in a straight line. It passes through three points, A, B and C with speeds $20\,\text{m s}^{-1}$, $30\,\text{m s}^{-1}$ and $45\,\text{m s}^{-1}$ respectively. The time taken to move from A to B is t_1 seconds and the time taken to move from B to C is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$.

Given also that the total time taken for the particle to move from A to C is 50 s,

b find the distance between A and B.

2.2 You can use the formulae $v^2 = u^2 + 2as$, $s = ut + \frac{1}{2}at^2$ and $s = vt - \frac{1}{2}at^2$ for a particle moving in a straight line with constant acceleration.

You can eliminate t from the formulae for constant acceleration.

$$t = \frac{v - u}{a}$$

Rearrange the formula $v = u + at$ to make t the subject.

$$s = \left(\frac{u + v}{2}\right)\left(\frac{v - u}{a}\right)$$

Substitute this expression for t into $s = \left(\frac{u + v}{2}\right)t$.

$$2as = v^2 - u^2$$

Multiply out the brackets and rearrange.

■ $v^2 = u^2 + 2as$

You can also eliminate v from the formulae for constant acceleration.

$$s = \left(\frac{u + u + at}{2}\right)t$$

Substitute $v = u + at$ into $s = \left(\frac{u + v}{2}\right)t$.

$$= \left(\frac{2u}{2} + \frac{at}{2}\right)t$$

$$= \left(u + \frac{1}{2}at\right)t$$

Multiply out the brackets and rearrange.

■ $\mathbf{s = ut + \frac{1}{2}at^2}$

Finally, you can eliminate u by substituting into this formula:

$$s = (v - at)t + \frac{1}{2}at^2$$

Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$.

■ $\mathbf{s = vt - \frac{1}{2}at^2}$

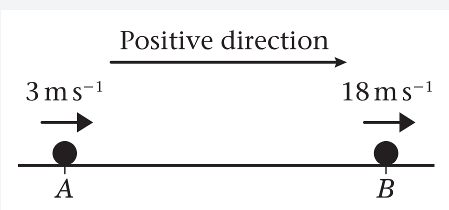
■ **You need to remember the five formulae for solving problems about particles moving in a straight lines with constant acceleration.**

- $v = u + at$
- $s = \left(\frac{u + v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

You need to remember these formulae. They are not given in the formula booklet in your exam.

Example 5

A particle is moving along a straight line from A to B with constant acceleration 5 m s^{-2}. The velocity of the particle at A is 3 m s^{-1} in the direction $\overrightarrow{AB}$. The velocity of the particle at B is 18 m s^{-1} in the same direction. Find the distance from A to B.



$a = 5,\ u = 3,\ v = 18,\ s = ?$

Write down the values you know and the values you need to find. This will help you choose the correct formula.

$v^2 = u^2 + 2as$

t is not involved so choose the formula that does not have t in it.

$18^2 = 3^2 + 2 \times 5 \times s$

$324 = 9 + 10s$

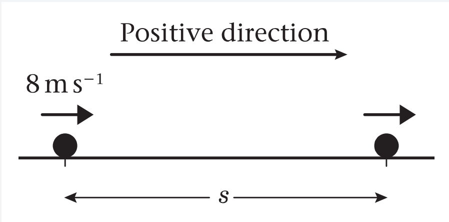
$s = \frac{324 - 9}{10} = 31.5$

Substitute in the values you are given and solve the equation for s. This gives the distance you were asked to find.

$AB = 31.5\text{ m}$

Example 6

A car is travelling along a straight horizontal road with a constant acceleration of $0.75\,\text{m}\,\text{s}^{-2}$. The car is travelling at $8\,\text{m}\,\text{s}^{-1}$ when it passes a pillar box. 12 seconds later it passes a lamp post. Find **a** the distance between the pillar box and the lamp post, **b** the speed with which the car passes the lamp post.



a $a = 0.75,\ u = 8,\ t = 12,\ s = ?$

$s = ut + \frac{1}{2}at^2$

$= 8 \times 12 + \frac{1}{2} \times 0.75 \times 12^2$

$= 96 + 54 = 150$

The distance between the pillar box and the lamp post is 150 m.

You are given a, u and t and asked to find s. The final velocity, v, is not given or asked for, so choose the formula without v.

b $a = 0.75,\ u = 8,\ t = 12,\ v = ?$

$v = u + at$

$= 8 + 0.75 \times 12$

$= 17\,\text{m}\,\text{s}^{-1}$

The speed of the car at the lamp post is $17\,\text{m}\,\text{s}^{-1}$.

In part **b**, you can use the same values for a, u and t but you are now asked to find v. In this part s is not needed, so choose the formula without s.

You could also solve part **b** using the value of s found in part **a** and the formula $v^2 = u^2 + 2as$.

There is an element of risk in using $s = 150$. This is your answer to part **a** and everyone makes mistakes from time to time. In examinations, it is a good plan to use the data given in a question, rather than your own answer, unless this is impossible or causes a lot of extra work.

Example 7

A particle moves with constant acceleration $1.5\,\text{m}\,\text{s}^{-2}$ in a straight line from a point A to a point B, where $AB = 16\,\text{m}$. At A, the particle has speed $3\,\text{m}\,\text{s}^{-1}$. Find the speed of the particle at B.

$a = 1.5,\ u = 3,\ s = 16,\ v = ?$

$v^2 = u^2 + 2as$

$= 3^2 + 2 \times 1.5 \times 16$

$v = \sqrt{57} \approx 7.5498\ldots$

The speed of the particle at B is $7.55\,\text{m}\,\text{s}^{-1}$, to three significant figures.

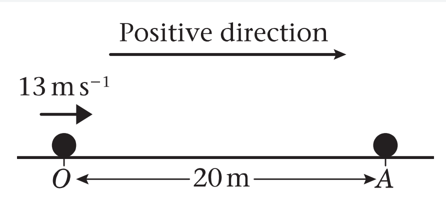
There is no t here, so choose the formula without t.

$v^2 = 57$ has two possible solutions, $v = \pm\sqrt{57}$. Look at the question to decide whether you need the positive or negative solution (or both). The particle has positive acceleration and positive initial speed, so v must be positive.

It is usually reasonable to give your answer to three significant figures.

Example 8

A particle is moving in a straight horizontal line with constant deceleration $4\,\text{m s}^{-2}$. At time $t = 0$ the particle passes through a point O with speed $13\,\text{m s}^{-1}$ travelling towards a point A where $OA = 20\,\text{m}$. Find **a** the times when the particle passes through A, **b** the velocities of the particle when it passes through A, **c** the values of t when the particle returns to O.



a $a = -4,\ u = 13,\ s = 20,\ t = ?$

> The particle is decelerating so the value of a is negative.

$s = ut + \frac{1}{2}at^2$

> You are told the values of a, u and s and asked to find t. You are given no information about v and are not asked to find it so you choose the formula without v.

$20 = 13t - \frac{1}{2} \times 4t^2$

$= 13t - 2t^2$

$2t^2 - 13t + 20 = 0$

> This is a quadratic equation. You can solve it using factorisation, or by using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$(2t - 5)(t - 4) = 0$

$t = \frac{5}{2}, 4$

> There are two answers. Both are correct. The particle moves from O to A, goes beyond A and then turns round and returns to A.

The particle moves through A twice, $2\frac{1}{2}$ seconds and 4 seconds after moving through O.

b $u = 13,\ a = -4,\ t = \frac{5}{2},\ v = ?$

$v = u + at$

> There are two values of t and you have to find the velocity of both. The formula $v = u + at$ is the simplest one to use.

$= 13 - 4 \times \frac{5}{2}$

$= 3$

> This answer is positive, so the particle is moving in the positive direction (away from O).

$u = 13,\ a = -4,\ t = 4,\ v = ?$

$v = u + at$

$= 13 - 4 \times 4$

$= -3$

> The value of v is negative when $t = 4$, so the particle is moving in the negative direction (towards O).

When $t = \frac{5}{2}$, the particle passes through A with velocity $3\,\text{m s}^{-1}$ in the direction $\overrightarrow{OA}$.

When $t = 4$, the particle passes through A with velocity $3\,\text{m s}^{-1}$ in the direction $\overrightarrow{AO}$.

> Remember a velocity has a direction as well as a magnitude. Velocity is a vector quantity. When you are asked for a velocity, your answer must contain a direction as well as a magnitude.

c The particle returns to O when $s = 0$.

$s = 0$, $u = 13$, $a = -4$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 13t - 2t^2$$

$$= t(13 - 2t)$$

$$t = 0, \frac{13}{2}$$

The particle returns to O 6.5 seconds after it first passed through O.

When the particle returns to O, its displacement (distance) from O is zero.

The first solution ($t = 0$) represents the starting position of the particle. The other solution ($t = \frac{13}{2}$) tells you when the particle returns to O.

Example 9

A cyclist is moving along a straight road with constant acceleration. She first passes a shop and 10 seconds later, travelling at $8\,\text{m s}^{-1}$, she passes a street sign. The distance between the shop and the street sign is 60 m. Find **a** the acceleration of the cyclist, **b** the speed with which she passed the shop.

a $t = 10$, $v = 8$, $s = 60$, $a = ?$

$$s = vt - \frac{1}{2}at^2$$

$$60 = 8 \times 10 - \frac{1}{2} \times a \times 100$$

$$60 = 80 - 50a$$

$$50a = 80 - 60 = 20$$

$$a = \frac{20}{50} = 0.4$$

The acceleration of the cyclist is $0.4\,\text{m s}^{-2}$.

b $v = 8$, $t = 10$, $a = 0.4$, $u = ?$

$$v = u + at$$

$$8 = u + 10 \times 0.4$$

$$u = 8 - 10 \times 0.4 = 4$$

The cyclist passes the shop with speed $4\,\text{m s}^{-1}$.

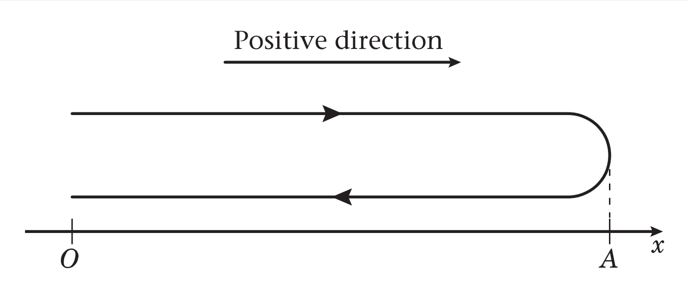
There is no u, so you choose the formula without u.

Substitute into the formula and solve the equation for a.

$v = u + at$ has been chosen as it is a simple formula. You could also use $s = \left(\frac{u + v}{2}\right)t$ which would avoid using your answer to part **a**.

Example 10

A particle P is moving on the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$, the particle P passes through the origin O, moving in the positive direction of x with speed 15 m s^{-1}. Find **a** the time between the instant when P first passes through O and the instant when it returns to O, **b** the total distance travelled by P during this time.



Before you start, draw a sketch so you can see what is happening. The particle moves through O with a positive velocity. As it is decelerating it slows down and will eventually have zero velocity at a point A, which you don't yet know. As the particle is still decelerating, its velocity becomes negative, so the particle changes direction and returns to O.

a $a = -2.5,\ u = 15,\ s = 0,\ t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 15t - \frac{1}{2} \times 2.5 \times t^2$$

$$0 = 60t - 5t^2$$

$$= 5t(12 - t)$$

$$t = 0,\ t = 12$$

The particle P returns to O after 12 s.

When the particle returns to O, its displacement (distance) from O is zero.

Multiply by 4 to get whole-number coefficients.

b $a = -2.5,\ u = 15,\ v = 0,\ s = ?$

$$v^2 = u^2 + 2as$$

$$0^2 = 15^2 - 2 \times 2.5 \times s$$

$$5s = 15^2 = 225$$

$$s = \frac{225}{5} = 45$$

The distance $OA = 45$ m.

The total distance travelled by P is $2 \times 45\text{ m} = 90\text{ m}$.

At the furthest point from O, labelled A in the diagram, the particle changes direction. At that point, for an instant, the particle has zero velocity.

In the 12 s the particle has been moving it has travelled to A and back. The total distance travelled is twice the distance OA.

Exercise 2B

1 A particle is moving in a straight line with constant acceleration 2.5 m s^{-2}. It passes a point A with speed 3 m s^{-1} and later passes through a point B, where $AB = 8$ m. Find the speed of the particle as it passes through B.

2 A car is accelerating at a constant rate along a straight horizontal road. Travelling at 8 m s^{-1}, it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.

3 A cyclist travelling at $12\,\text{m s}^{-1}$ applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

4 A particle moves along a straight line from P to Q with constant acceleration $1.5\,\text{m s}^{-2}$. The particle takes 4 s to pass from P to Q and $PQ = 22$ m. Find the speed of the particle at Q.

5 A particle is moving along a straight line OA with constant acceleration $2\,\text{m s}^{-2}$. At O the particle is moving towards A with speed $5.5\,\text{m s}^{-1}$. The distance OA is 20 m. Find the time the particle takes to move from O to A.

6 A train is moving along a straight horizontal track with constant acceleration. The train passes a signal at $54\,\text{km h}^{-1}$ and a second signal at $72\,\text{km h}^{-1}$. The distance between the two signals is 500 m. Find, in m s^{-2}, the acceleration of the train.

7 A particle moves along a straight line, with constant acceleration, from a point A to a point B where $AB = 48$ m. At A the particle has speed $4\,\text{m s}^{-1}$ and at B it has speed $16\,\text{m s}^{-1}$. Find **a** the acceleration of the particle, **b** the time the particle takes to move from A to B.

8 A particle moves along a straight line with constant acceleration $3\,\text{m s}^{-2}$. The particle moves 38 m in 4 s. Find **a** the initial speed of the particle, **b** the final speed of the particle.

9 The driver of a car is travelling at $18\,\text{m s}^{-1}$ along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of $3\,\text{m s}^{-2}$. Find **a** the distance travelled as the car decelerates, **b** the time it takes for the car to decelerate from $18\,\text{m s}^{-1}$ to rest.

10 A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is $12\,\text{m s}^{-1}$. The friction between the stone and the ice causes the stone to slow down at a constant rate of $0.8\,\text{m s}^{-2}$. Find **a** the distance moved by the stone before coming to rest, **b** the speed of the stone at the instant when it has travelled half of this distance.

11 A particle is moving along a straight line OA with constant acceleration $2.5\,\text{m s}^{-2}$. At time $t = 0$, the particle passes through O with speed $8\,\text{m s}^{-1}$ and is moving in the direction OA. The distance OA is 40 m. Find **a** the time taken for the particle to move from O to A, **b** the speed of the particle at A. Give your answers to one decimal place.

12 A particle travels with uniform deceleration $2\,\text{m s}^{-2}$ in a horizontal line. The points A and B lie on the line and $AB = 32$ m. At time $t = 0$, the particle passes through A with velocity $12\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$. Find **a** the values of t when the particle is at B, **b** the velocity of the particle for each of these values of t.

13 A particle is moving along the x-axis with constant deceleration $5\,\text{m s}^{-2}$. At time $t = 0$, the particle passes through the origin O with velocity $12\,\text{m s}^{-1}$ in the positive direction. At time t seconds the particle passes through the point A with x-coordinate 8. Find **a** the values of t, **b** the velocity of the particle as it passes through the point with x-coordinate -8.

14 A particle P is moving on the x-axis with constant deceleration $4\,\text{m}\,\text{s}^{-2}$. At time $t = 0$, P passes through the origin O with velocity $14\,\text{m}\,\text{s}^{-1}$ in the positive direction. The point A lies on the axis and $OA = 22.5\,\text{m}$. Find **a** the difference between the times when P passes through A, **b** the total distance travelled by P during the interval between these times.

15 A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points A, B and C where $AB = 100\,\text{m}$ and $BC = 300\,\text{m}$. The speed of the car at B is $14\,\text{m}\,\text{s}^{-1}$ and the speed of the car at C is $20\,\text{m}\,\text{s}^{-1}$. Find **a** the acceleration of the car, **b** the time take for the car to travel from A to C.

16 Two particles P and Q are moving along the same straight horizontal line with constant accelerations $2\,\text{m}\,\text{s}^{-2}$ and $3.6\,\text{m}\,\text{s}^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $4\,\text{m}\,\text{s}^{-1}$. One second later Q passes through A with speed $3\,\text{m}\,\text{s}^{-1}$, moving in the same direction as P.

a Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A.

b Find the value of t where the particles meet.

c Find the distance of A from the point where the particles meet.

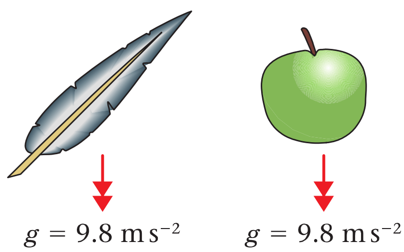
2.3 You can use the formulae for constant acceleration to model an object moving vertically in a straight line under gravity.

- **The force of gravity causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.**

  This means that in a vacuum an apple and a feather would both accelerate downwards at the same rate.

- **On earth, the acceleration due to gravity is represented by the letter g and is approximately $9.8\,\text{m}\,\text{s}^{-2}$. The actual value of the acceleration can vary by very small amounts in different places due to the changing radius of the Earth and height above sea level.**

In M1 you will always use $g = 9.8\,\text{m}\,\text{s}^{-2}$. This is an approximation to two significant figures. If you use this value in your working you should give your answer to the same degree of accuracy.



Objects falling in a vacuum accelerate at the same rate regardless of their mass or size.

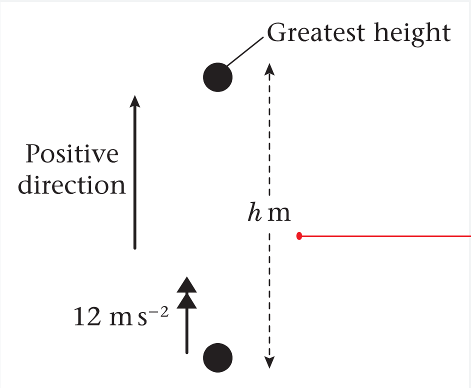
- **An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\,\text{m}\,\text{s}^{-2}$**

- **When solving problems about vertical motion you can choose the positive direction to be either upwards or downwards. Acceleration due to gravity is always downwards, so if the positive direction is upwards then $a = -9.8\,\text{m}\,\text{s}^{-2}$.**

- **The total time that an object is in motion from the time it is projected (thrown) upwards to the time it hits the ground is called the time of flight. The initial speed is sometimes called the speed of projection.**

Example 11

A ball B is projected vertically upwards from a point O with speed $12\,\text{m}\,\text{s}^{-1}$. Find **a** the greatest height above O reached by B, **b** the total time before B returns to O.



a $u = 12$

$v = 0$

$a = -9.8$

$s = h$

$v^2 = u^2 + 2as$

$0^2 = 12^2 - 2 \times 9.8 \times h$

$$h = \frac{12^2}{2 \times 9.8} = \frac{144}{19.6} = 7.346\ldots$$

The greatest height above O reached by B is 7.4 m, to two significant figures.

b $s = 0$

$u = 12$

$a = -9.8$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 12t - \frac{1}{2} \times 9.8 \times t = t(12 - 4.9t)$$

$$t = \frac{12}{4.9} = 2.448\ldots$$

The time taken for B to return to O is 2.4 s, to two significant figures.

You first decide which direction you will take as positive. As the ball is projected upwards, you take the upwards direction as positive.

Writing the unknown is h metres helps you to remember you are finding a height!

At the highest point of its path, the ball is turning round. For an instant, its speed is zero.

The positive direction is upwards, and gravity acts downwards, so a is negative.

Write your answer correct to two significant figures.

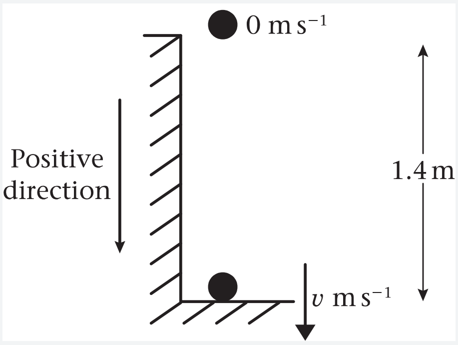
When the ball returns to its original position O, its displacement from O is zero.

This equation has two answers but one of them is $t = 0$. This represents the start of the ball's motion, so you want the other answer.

You must work to at least three significant figures and correct your answer to two significant figures.

Example 12

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find **a** the time the book takes to reach the floor, **b** the speed with which the book strikes the floor.



a $s = 1.4$

$a = +9.8$

$u = 0$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$1.4 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$t^2 = \frac{1.4}{4.9} = 0.2857...$

$t = \sqrt{0.2857...} = 0.5345...$

The time taken for the book to reach the floor is 0.53 s, to two significant figures.

b $s = 1.4$

$a = 9.8$

$u = 0$

$v = ?$

$v^2 = u^2 + 2as$

$= 0^2 + 2 \times 9.8 \times 1.4 = 27.44$

$v = \sqrt{27.44} = 5.238... \approx 5.2$

The book hits the floor with speed 5.2 m s^{-1}, to two significant figures.

Model the book as a particle moving in a straight line with a constant acceleration of magnitude 9.8 m s^{-2}.

As the book is moving downwards throughout its motion, it is sensible to take the downwards direction as positive.

You have taken the downwards direction as positive and gravity acts downwards. Here the acceleration is positive.

Assume the book has an initial speed of zero.

Choose the formula without v.

Solve the equation for t^2 and use your calculator to find the positive square root.

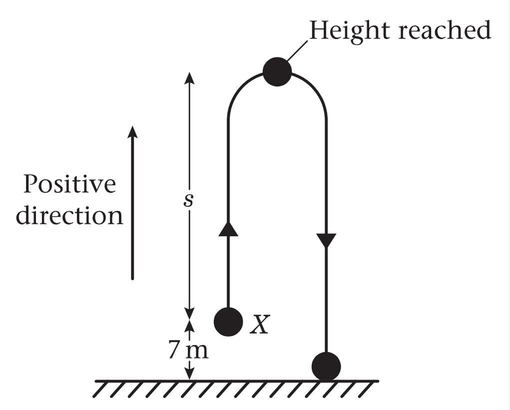
Remember to give the answer to two significant figures.

Choose the formula without t.

Remember to show your working to at least three significant figures. You can use unrounded values in your calculations by using the **Ans** button on your calculator.

Example 13

A ball is projected vertically upwards, from a point X which is 7 m above the ground, with speed 21 m s^{-1}. Find **a** the greatest height above the ground reached by the ball, **b** the time of flight of the ball.



In this sketch the upward and downwards motion have been sketched side by side. In reality they would be on top of one another, but drawing them separately makes it easier to see what is going on.

You model the ball as a particle moving in a straight line with a constant acceleration of magnitude 9.8 m s^{-2}.

a $u = 21$

$v = 0$

$a = -9.8$

$s = ?$

$v^2 = u^2 + 2as$

$0^2 = 21^2 + 2 \times (-9.8) \times s = 441 - 19.6s$

$s = \frac{441}{19.6} = 22.5$

$(22.5 + 7)\text{ m} = 29.5\text{ m}$

The greatest height reached by the ball above the ground is 30 m, to two significant figures.

At its highest point, the ball is turning round. For an instant, it is neither going up or down, so its speed is zero.

22.5 m is the distance the ball has moved above X but X is 7 m above the ground. You must add on another 7 m to get the greatest height above the ground reached by the ball.

b $s = -7$

$u = 21$

$a = -9.8$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$-7 = 21t - 4.9t^2$

$4.9t^2 - 21t - 7 = 0$

$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{-(-21) \pm \sqrt{((-21)^2 - 4 \times 4.9 \times (-7))}}{2 \times 4.9}$$

$$= \frac{21 \pm \sqrt{578.2}}{9.8} \approx \frac{21 \pm 24.046}{9.8}$$

$$\approx 4.5965, -0.3108$$

The time of flight of the ball is 4.6 s, to two significant figures.

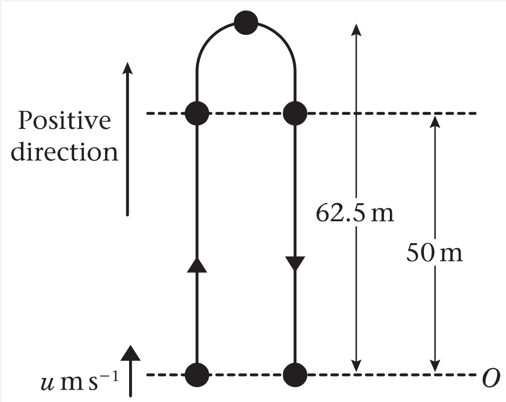
The time of flight is the total time that the ball is in motion from the time that it is projected to the time that it stops moving. Here the ball will stop when it hits the ground. The point where the ball hits the ground is 7 m **below** the point from which it was projected so $s = -7$.

Rearrange the equation and use the quadratic formula.

The negative answer represents a time before the particle was projected, so you need the positive answer. Remember to give your answer to two significant figures.

Example 14

A particle is projected vertically upwards from a point O with speed $u\,\text{m s}^{-1}$. The greatest height reached by the particle is 62.5 m above O. Find **a** the value of u, **b** the total time for which the particle is 50 m or more above O.



The particle will pass through the point 50 m above O twice. Once on the way up and once on the way down.

a $v = 0$

$s = 62.5$

$a = -9.8$

$u = ?$

There is no t, so you choose the formula without t.

$v^2 = u^2 + 2as$

$0^2 = u^2 - 2 \times 9.8 \times 62.5$

$u^2 = 1225$

$u = \sqrt{1225} = 35$

In this part, you obtain an exact answer, so there is no need for approximation.

b $s = 50$

$u = 35$

$a = -9.8$

$t = ?$

Two values of t need to be found: one on the way up and one on the way down.

$s = ut + \frac{1}{2}at^2$

$50 = 35t - 4.9t^2$

$4.9t^2 - 35t + 50 = 0$

Write this equation in the form $ax^2 + bx + c = 0$ and use the quadratic formula.

$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{35 \pm \sqrt{(35^2 - 4 \times 4.9 \times 50)}}{9.8}$$

$$= \frac{35 \pm \sqrt{245}}{9.8} = \frac{35 \pm 15.6525}{9.8}$$

$= 5.1686\ldots, 1.9742\ldots$

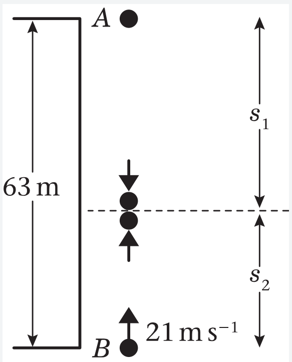
$(5.1686\ldots) - (1.9742\ldots) \approx 3.194$

Between these two times the particle is always more than 50 m above O. You find the total time for which the particle is 50 m or more above O by finding the difference of these two values.

The total time for which the particle is 50 m or more above O is 3.2 m, to two significant figures.

Example 15

A ball A falls vertically from rest from the top of a tower 63 m high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed 21 m s^{-1}. The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.



You must take special care with problems where objects are moving in different directions. Here A is moving downwards and you will take the acceleration due to gravity as positive. However B is moving upwards so for B the acceleration due to gravity is negative.

For A, the motion is downwards

$u = 0$

$a = 9.8$

$s = ut + \frac{1}{2}at^2$

$s_1 = 4.9t^2$

You cannot find s_1 at this stage. You have to express it in terms of t.

For B, the motion is upwards

$u = 21$

$a = -9.8$

As B is moving upwards, the acceleration due to gravity is negative.

$s = ut + \frac{1}{2}at^2$

$s_2 = 21t - 4.9t^2$

You now have expressions for s_1 and s_2 in terms of t.

The height of the tower is 63 m.

$s_1 + s_2 = 63$

Adding together the two distances gives the height of the tower. You can write this as an equation in t.

$4.9t^2 + (21t - 4.9t^2) = 63$

The $4.9t^2$ and $-4.9t^2$ cancel.

$21t = 63$

$t = 3$

$s_2 = 21t - 4.9t^2$

$= 21 \times 3 - 4.9 \times 3^2 = 18.9$

You have found t but you were asked for the distance from the bottom of the tower. Substitute your value for t into your equation for s_2.

The balls collide 19 m from the bottom of the tower, to two significant figures.

Exercise 2C

1 A ball is projected vertically upwards from a point O with speed 14 m s^{-1}. Find the greatest height above O reached by the ball.

2 A well is 50 m deep. A stone is released from rest at the top of the well. Find how long the stone takes to reach the bottom of the well.

3 A book falls from the top shelf of a bookcase. It takes 0.6 s to reach the floor. Find how far it is from the top shelf to the floor.

4 A particle is projected vertically upwards with speed 20 m s^{-1} from a point on the ground. Find the time of flight of the particle.

5 A ball is thrown vertically downward from the top of a tower with speed 18 m s^{-1}. It reaches the ground in 1.6 s. Find the height of the tower.

6 A pebble is catapulted vertically upwards with speed 24 m s^{-1}. Find **a** the greatest height above the point of projection reached by the pebble, **b** the time taken to reach this height.

7 A ball is projected upwards from a point which is 4 m above the ground with speed 18 m s^{-1}. Find **a** the speed of the ball when it is 15 m above its point of projection, **b** the speed with which the ball hits the ground.

8 A particle P is projected vertically downwards from a point 80 m above the ground with speed 4 m s^{-1}. Find **a** the speed with which P hits the ground, **b** the time P takes to reach the ground.

9 A particle P is projected vertically upwards from a point X. Five seconds later P is moving downwards with speed 10 m s^{-1}. Find **a** the speed of projection of P, **b** the greatest height above X attained by P during its motion.

10 A ball is thrown vertically upwards with speed 21 m s^{-1}. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

11 A stone is thrown vertically upward from a point which is 3 m above the ground, with speed 16 m s^{-1}. Find **a** the time of flight of the stone, **b** the total distance travelled by the stone.

12 A particle is projected vertically upwards with speed 24.5 m s^{-1}. Find the total time for which it is 21 m or more above its point of projection.

13 A particle is projected vertically upwards from a point O with speed $u\text{ m s}^{-1}$. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u\text{ m s}^{-1}$. Find **a** the value of u, **b** the time from the instant that the particle leaves O to the instant that it returns to O.

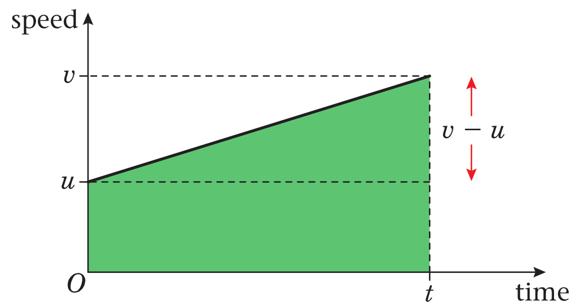
14 A ball A is thrown vertically downwards with speed 5 m s^{-1} from the top of a tower block 46 m above the ground. At the same time as A is thrown downwards, another ball B is thrown vertically upwards from the ground with speed 18 m s^{-1}. The balls collide. Find the distance of the point where A and B collide from the point where A was thrown.

15 A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball **a** the first time it rebounds from the floor, **b** the second time it rebounds from the floor.

16 A particle P is projected vertically upwards from a point O with speed $12\,\text{m s}^{-1}$. One second after P has been projected from O, another particle Q is projected vertically upwards from O with speed $20\,\text{m s}^{-1}$. Find **a** the time between the instant that P is projected from O and the instant when P and Q collide, **b** the distance of the point where P and Q collide from O.

2.4 You can represent the motion of an object on a speed–time graph or a distance–time graph

- **In a speed–time graph speed is always plotted on the vertical axis and time is always plotted on the horizontal axis. This speed–time graph represents the motion of a particle accelerating from speed *u* at time 0 to speed *v* at time *t*.**



Gradient of line $= \dfrac{\text{change of velocity}}{\text{time}}$

$$= \frac{v-u}{t} = a$$

So the gradient of the speed–time graph is the acceleration of the particle. If the line is straight the acceleration is constant.

Using the formula for the area of a trapezium:

Shaded area $= \left(\dfrac{u+v}{2}\right)t$

$$= s$$

So the area under the speed–time graph is the distance travelled by the particle.

- **The gradient of a speed–time graph is the acceleration.**

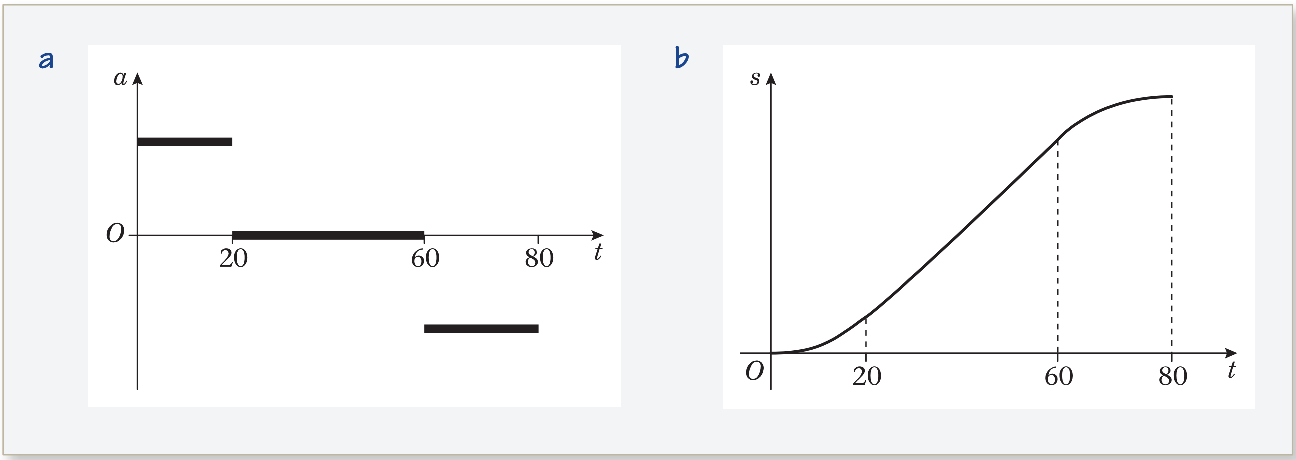
- **The area under a speed–time graph is the distance travelled.**

You can also draw acceleration–time graphs and distance–time graphs for the motion of a particle. Time is always plotted on the horizontal axis.

If a particle is moving with constant speed its distance–time graph will be a straight line. If it is accelerating or decelerating then its distance–time graph will be a curve.

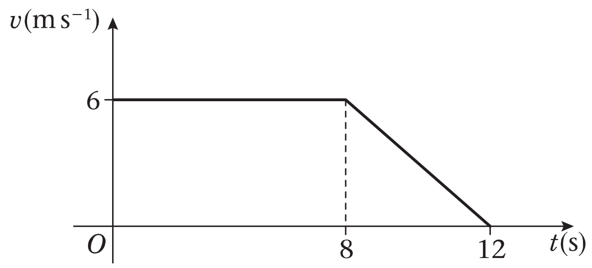
Example 16

A car accelerates uniformly from rest for 20 seconds. It travels at a constant speed for the next 40 seconds, then decelerates uniformly for 20 seconds until it is stationary. Sketch **a** an acceleration–time graph, **b** a distance–time graph for the motion of the car.



Example 17

The figure shows a speed–time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 s. For the first 8 s, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 s.



Find **a** the distance travelled by the cyclist during this 12 s period, **b** the rate at which the cyclist decelerates.

Model the cyclist as a particle moving in a straight line.

a The distance travelled is given by

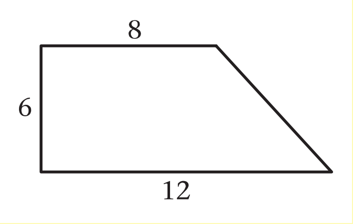
$$s = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(8 + 12) \times 6$$

$$= 10 \times 6 = 60$$

The distance travelled by the cyclist is 60 m.

The distance travelled is represented by the area of the trapezium with these sides.



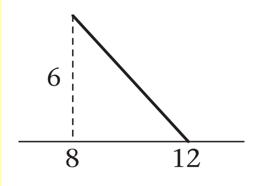
You can use the formula for the area of a trapezium to calculate this area.

b The acceleration is the gradient of the slope.

$$a = \frac{-6}{4} = -1.5$$

The deceleration is $1.5\,\text{m}\,\text{s}^{-2}$.

The gradient is given by
$$\frac{\text{the difference in the } v \text{ coordinates}}{\text{the difference in the } t \text{ coordinates}}$$



Here, the value of v **decreases** by 6 as t **increases** by 4.

Example 18

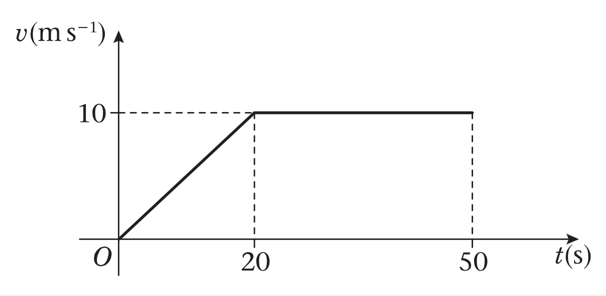
A car is waiting at traffic lights. When the lights turn green, the car accelerates uniformly from rest to a speed of $10\,\text{m}\,\text{s}^{-1}$ in 20 s. This speed is then maintained until the car passes a road sign 50 s after leaving the traffic lights.

a Sketch a speed–time graph to illustrate the motion of the car.

b Find the distance between the traffic lights and the road sign.

Model the car as a particle moving in a straight line.

a



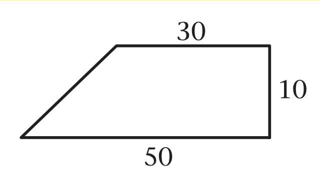
When you are asked to sketch a graph you should use ordinary paper and not graph paper. You should use a ruler but you do not have to draw lengths accurately to scale. You should label the axes and indicate any relevant information given in the question.

b The distance travelled is given by

$$s = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(30 + 50) \times 10$$
$$= 40 \times 10 = 400$$

The distance between the traffic lights and the road sign is 400 m.

The dimensions of the trapezium are:



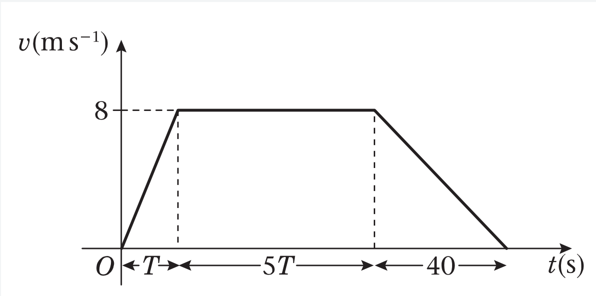
Example 19

A particle moves along a straight line. The particle accelerates uniformly from rest to a speed of $8\,\text{m s}^{-1}$ in T seconds. The particle then travels at a constant speed of $8\,\text{m s}^{-1}$ for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a speed–time graph to illustrate the motion of the particle.

Given that the total distance travelled by the particle is 600 m, **b** find the value of T, **c** sketch an acceleration–time graph illustrating the motion of the particle.

a



b The area of the trapezium is:

$s = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(5T + 6T + 40) \times 8$

The length of the shorter of the two parallel sides is $5T$. The length of the longer side is $T + 5T + 40 = 6T + 40$.

$= 4(11T + 40)$

The distance moved is 600 m.

$4(11T + 40) = 600$

The distance moved is equal to the area of the trapezium. Write an equation and solve it to find T.

$44T + 160 = 600$

$T = \frac{600 - 160}{44} = 10$

c The acceleration in the first 10 s is given by

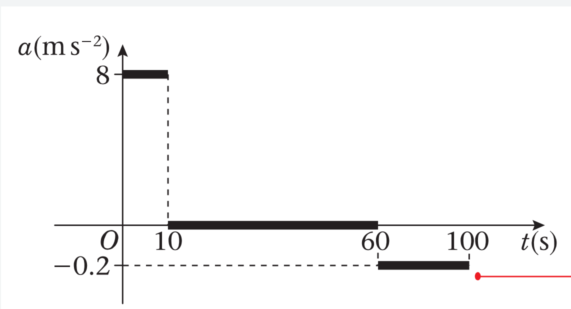
$a = \frac{8}{10} = 0.8.$

For the first ten seconds the v-coordinate **increases** by 8 as the t-coordinate increases by 10. This gives a positive answer.

The acceleration in the last 40 s is given by

$a = \frac{-8}{40} = -0.2$

In the last forty seconds the v-coordinate **decreases** by 8 as the t-coordinate increases by 40. This gives a negative answer.



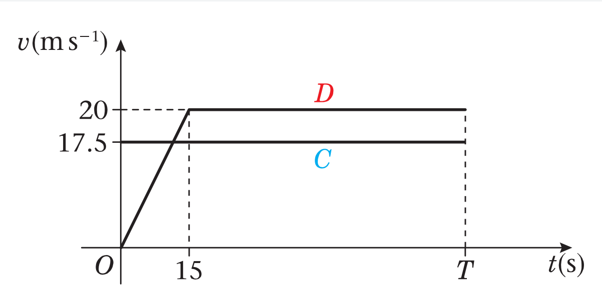
Example 20

A car C is moving along a straight road with constant speed $17.5\,\text{m s}^{-1}$. At time $t = 0$, C passes a lay-by. At time $t = 0$, a second car D leaves the lay-by. Car D accelerates from rest to a speed of $20\,\text{m s}^{-1}$ in 15 s and then maintains a constant speed of $20\,\text{m s}^{-1}$. Car D passes car C at a road sign.

a On the same diagram, sketch speed–time graphs to illustrate the motion of the two cars.

b Find the distance between the lay-by and the road sign.

a



You should label the lines so that it is clear which represents the motion of C and which represents the motion of D.

b Let D pass C at time $t = T$.

It is difficult to find the distance travelled directly. You can find the time the cars pass first. It does not matter what letter you choose for the time. T has been used here.

The distance travelled by C is given by

$s = 17.5T$

As C is travelling at a constant speed you use the formula distance = speed × time.

The distance travelled by D is given by

$$s = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(T - 15 + T) \times 20$$

$$= 10(2T - 15)$$

The longer of the parallel sides of the trapezium is T. The shorter of the parallel sides is $(T - 15)$.

The distances travelled by C and D are the same.

$$10(2T - 15) = 17.5T$$

$$20T - 150 = 17.5T$$

$$2.5T = 150$$

$$T = \frac{150}{2.5} = 60$$

As the cars were at the lay-by at the same time and the road sign at the same time, the distance travelled by both of them is the same. You equate the distances to get an equation in T and solve it.

$s = 17.5T = 17.5 \times 60 = 1050$

To find the distance travelled you can substitute into the expression for the distance travelled by either C or by D.

The distance from the lay-by to the road sign is 1050 m.

Example 21

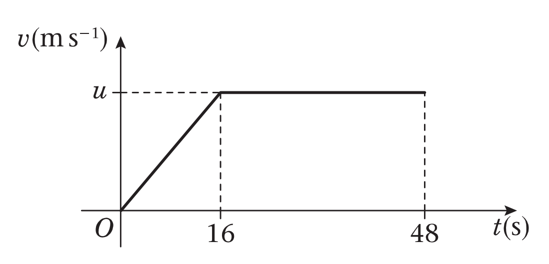
A particle is moving along a horizontal axis Ox. At time $t = 0$, the particle is at rest at O. The particle then accelerates at a constant rate, reaching a speed of $u\,\text{m s}^{-1}$ in 16 s. The particle maintains the speed of $u\,\text{m s}^{-1}$ for a further 32 s. After 48 s, the particle is at a point A, where $OA = 320$ m.

a Sketch a speed–time graph to illustrate the motion of the particle.

b Find the value of u.

c Sketch a distance–time graph for the particle.

a



b $\frac{1}{2}(32 + 48) \times u = 320$

$40u = 320$

$u = 8$

The area of the trapezium is 320. You use this to obtain an equation in u, which you solve.

c The gradient of the line in the first 16 s is the acceleration and is given by

$a = \frac{u}{16} = \frac{8}{16} = \frac{1}{2}$

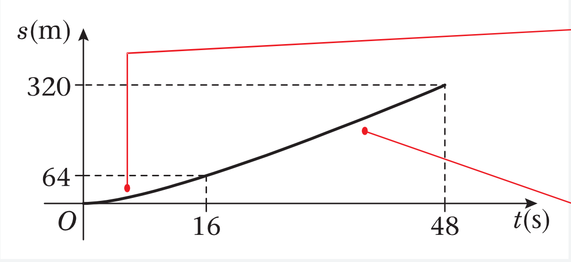
Use your answer from part **b** and the property that the gradient of a speed–time graph is the acceleration to find the value of the acceleration.

$s = ut + \frac{1}{2}at^2$

$= \frac{1}{4}t^2$

The initial speed is zero and the acceleration is $\frac{1}{2}$.

When $t = 16$, $s = \frac{1}{4} \times 16^2 = 64$.

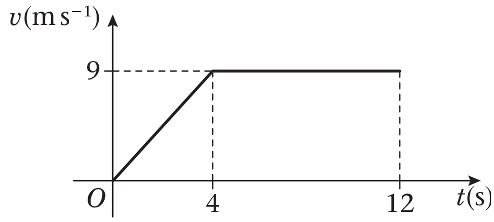


For the first 16 s the distance–time graph is a curve. It is part of the parabola with equation $s = \frac{1}{4}t^2$.

For the following 32 s, the particle moves with constant speed and the graph is a straight line.

Exercise 2D

1



The diagram shows the speed–time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a speed of 9 m s^{-1}. This speed is then maintained for a further 8 s. Find

a the rate at which the athlete accelerates,

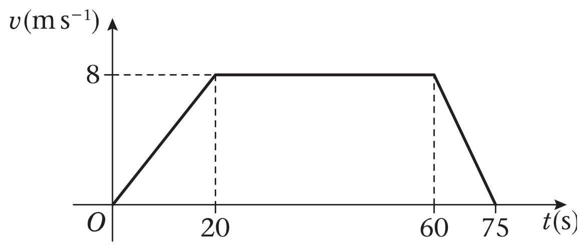
b the total distance travelled by the athlete in 12 s.

2 A car is moving along a straight road. When $t = 0$ s, the car passes a point A with speed 10 m s^{-1} and this speed is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.

a Sketch a speed–time graph to illustrate the motion of the car.

b Find the distance from A to B.

3



The diagram shows the speed–time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s^{-1} in 20 s. She then travels at a constant speed of 8 m s^{-1} for 40 s. She then decelerates uniformly to rest in 15 s. Find

a the acceleration of the cyclist in the first 20 s of motion,

b the deceleration of the cyclist in the last 15 s of motion,

c the total distance travelled in 75 s.

4 A car accelerates at a constant rate, starting from rest at a point A and reaching a speed of 45 km h^{-1} in 20 s. This speed is then maintained and the car passes a point B 3 minutes after leaving A.

a Sketch a speed–time graph to illustrate the motion of the car.

b Find the distance from A to B.

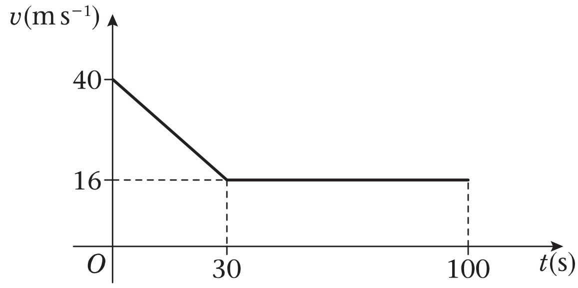
5 A motorcyclist starts from rest at a point S on a straight race track. He moves with constant acceleration for 15 s, reaching a speed of 30 m s^{-1}. He then travels at a constant speed of 30 m s^{-1} for T seconds. Finally he decelerates at a constant rate coming to rest at a point F, 25 s after he begins to decelerate.

a Sketch a speed–time graph to illustrate the motion.

Given that the distance between S and F is 2.4 km,

b calculate the time the motorcyclist takes to travel from S to F.

6



A train is travelling along a straight track. To obey a speed restriction, the brakes of the train are applied for 30 s reducing the speed of the train from 40 m s^{-1} to 16 m s^{-1}. The train then continues at a constant speed of 16 m s^{-1} for a further 70 s. The diagram shows a speed–time graph illustrating the motion of the train for the total period of 100 s. Find

a the retardation of the train in the first 30 s.

b the total distance travelled by the train in 100 s.

7 A train starts from a station X and moves with constant acceleration 0.6 m s^{-2} for 20 s. The speed it has reached after 20 s is then maintained for T seconds. The train then decelerates from this speed to rest in a further 40 s, stopping at a station Y.

a Sketch a speed–time graph to illustrate the motion of the train.

Given that the distance between the stations is 4.2 km, find

b the value of T,

c the distance travelled by the train while it is moving with constant speed.

8 A particle moves along a straight line. The particle accelerates from rest to a speed of 10 m s^{-1} in 15 s. The particle then moves at a constant speed of 10 m s^{-1} for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant speed is 4 times the period of time for which it is decelerating.

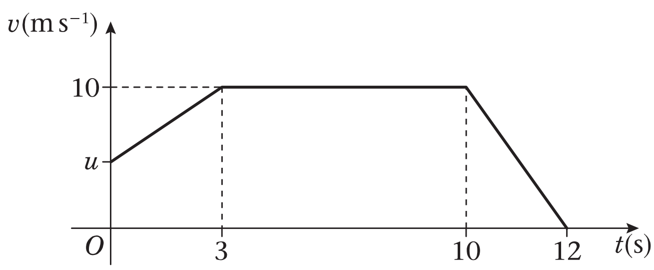
a Sketch a speed–time graph to illustrate the motion of the particle.

Given that the total distance travelled by the particle is 480 m,

b find the total time for which the particle is moving,

c sketch an acceleration–time graph illustrating the motion of the particle.

9

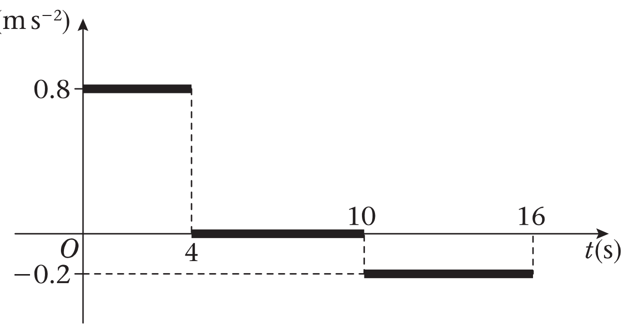


A particle moves 100 m in a straight line. The diagram is a sketch of a speed–time graph of the motion of the particle. The particle starts with speed u m s^{-1} and accelerates to a speed 10 m s^{-1} in 3 s. The speed of 10 m s^{-1} is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find

a the value of u,

b the acceleration of the particle in the first 3 s of motion.

10



The diagram is an acceleration–time graph to show the motion of a particle. At time $t = 0$ s, the particle is at rest. Sketch a speed–time graph for the motion of the particle.

11 A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates at a rate of $3\,\text{m s}^{-2}$ for 8 s and then maintains the speed she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to $30\,\text{m s}^{-1}$ in 20 s and then maintains the speed of $30\,\text{m s}^{-1}$. C passes M as they both pass a pedestrian.

a On the same diagram, sketch speed–time graphs to illustrate the motion of M and C.

b Find the distance of the pedestrian from the road junction.

12 A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant speed $15\,\text{m s}^{-1}$. The particle then decelerates from $15\,\text{m s}^{-1}$ to rest in T seconds.

a Sketch a speed–time graph to illustrate the motion of the particle.

The total distance travelled by the particle is 570 m.

b Find the value of T.

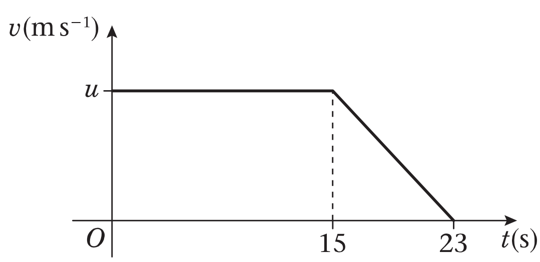
c Sketch a distance–time graph illustrating the motion of the particle.

Mixed exercise 2E

1 A car travelling along a straight road at $14\,\text{m s}^{-1}$ is approaching traffic lights. The driver applies the brakes and the car comes to rest with constant deceleration. The distance from the point where the brakes are applied to the point where the car comes to rest is 49 m. Find the deceleration of the car.

2 A ball is thrown vertically downward from the top of a tower with speed $6\,\text{m s}^{-1}$. The ball strikes the ground with speed $25\,\text{m s}^{-1}$. Find the time the ball takes to move from the top of the tower to the ground.

3



The diagram is a speed–time graph representing the motion of a cyclist along a straight road. At time $t = 0$ s, the cyclist is moving with speed $u\,\text{m s}^{-1}$. The speed is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s. The total distance she travels in 23 s is 152 m. Find the value of u.

4 A stone is projected vertically upwards with speed 21 m s^{-1}. Find

a the greatest height above the point of projection reached by the stone,

b the time between the instant that the stone is projected and the instant that it reaches its greatest height.

5 A train is travelling with constant acceleration along a straight track. At time $t = 0$ s, the train passes a point O travelling with speed 18 m s^{-1}. At time $t = 12$ s, the train passes a point P travelling with speed 24 m s^{-1}. At time $t = 20$ s, the train passes a point Q. Find

a the speed of the train at Q,

b the distance from P to Q.

6 A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with speed 40 km h^{-1} and a post box with speed of 24 km h^{-1}. The distance between the road sign and the post box is 240 m. Find, in m s^{-2}, the deceleration of the car.

7 A skier is travelling downhill along a straight path with constant acceleration. At time $t = 0$ s, she passes a point A with speed 6 m s^{-1}. She continues with the same acceleration until she reaches a point B with speed 15 m s^{-1}. At B, the path flattens out and she travels from B to a point C at the constant speed of 15 m s^{-1}. It takes 20 s for the skier to travel from B to C and the distance from A to C is 615 m.

a Sketch a speed–time graph to illustrate the motion of the skier.

b Find the distance from A to B.

c Find the time the skier took to travel from A to B.

8 A child drops a ball from a point at the top of a cliff which is 82 m above the sea. The ball is initially at rest. Find

a the time taken for the ball to reach the sea,

b the speed with which the ball hits the sea.

c State one physical factor which has been ignored in making your calculation.

9 A particle moves along a straight line, from a point X to a point Y, with constant acceleration. The distance from X to Y is 104 m. The particle takes 8 s to move from X to Y and the speed of the particle at Y is 18 m s^{-1}. Find

a the speed of the particle at X,

b the acceleration of the particle.

The particle continues to move with the same acceleration until it reaches a point Z. At Z the speed of the particle is three times the speed of the particle at X.

c Find the distance XZ.

10 A pebble is projected vertically upwards with speed 21 m s^{-1} from a point 32 m above the ground. Find

a the speed with which the pebble strikes the ground,

b the total time for which the pebble is more than 40 m above the ground.

11 A particle P is moving along the x-axis with constant deceleration $2.5\,\text{m s}^{-2}$. At time $t = 0\,\text{s}$, P passes through the origin with velocity $20\,\text{m s}^{-1}$ in the direction of x increasing. At time $t = 12\,\text{s}$, P is at the point A. Find

a the distance OA,

b the total distance P travels in 12 s.

12 A train starts from rest at a station P and moves with constant acceleration for 45 s reaching a speed of $25\,\text{m s}^{-1}$. The train then maintains this speed for 4 minutes. The train then uniformly decelerates, coming to rest at a station Q.

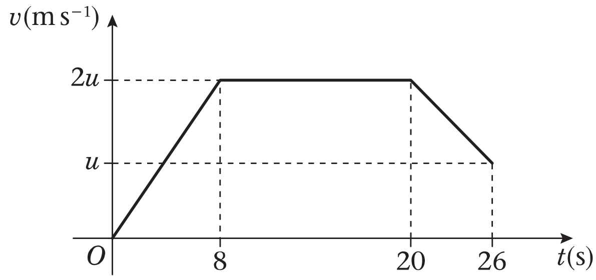
a Sketch a speed–time graph illustrating the motion of the train from P to Q.

The distance between the stations is 7 km.

b Find the deceleration of the train.

c Sketch an acceleration–time graph illustrating the motion of the train from P to Q.

13



A particle moves 451 m in a straight line. The diagram shows a speed–time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u\,\text{m s}^{-1}$ at time $t = 26\,\text{s}$. Find

a the value of u,

b the distance moved by the particle while its speed is less than $u\,\text{m s}^{-1}$.

14 A particle is moving in a straight line. The particle starts with speed $5\,\text{m s}^{-1}$ and accelerates at a constant rate of $2\,\text{m s}^{-1}$ for 8 s. It then decelerates at a constant rate coming to rest in a further 12 s.

a Sketch a speed–time graph illustrating the motion of the particle.

b Find the total distance moved by the particle during its 20 s of motion.

c Sketch a distance–time graph illustrating the motion of the particle.

15 A boy projects a ball vertically upwards with speed $10\,\text{m s}^{-1}$ from a point X, which is 50 m above the ground. T seconds after the first ball is projected upwards, the boy drops a second ball from X. Initially the second ball is at rest. The balls collide 25 m above the ground. Find the value of T.

16 A car is moving along a straight road with uniform acceleration. The car passes a check-point A with speed $12\,\text{m s}^{-1}$ and another check-point C with speed $32\,\text{m s}^{-1}$. The distance between A and C is 1100 m.

a Find the time taken by the car to move from A to C.

Given that B is the mid-point of AC,

b find the speed with which the car passes B.

17 A particle is projected vertically upwards with a speed of $30\,\text{m s}^{-1}$ from a point A. The point B is h metres above A. The particle moves freely under gravity and is above B for a time 2.4 s. Calculate the value of h.

18 Two cars A and B are moving in the same direction along a straight horizontal road. At time $t = 0$, they are side by side, passing a point O on the road. Car A travels at a constant speed of $30\,\text{m s}^{-1}$. Car B passes O with a speed of $20\,\text{m s}^{-1}$, and has constant acceleration of $4\,\text{m s}^{-2}$. Find

a the speed of B when it has travelled 78 m from O,

b the distance from O of A when B is 78 m from O,

c the time when B overtakes A.

19 A car is being driven on a straight stretch of motorway at a constant speed of $34\,\text{m s}^{-1}$, when it passes a speed restriction sign S warning of road works ahead and requiring speeds to be reduced to $22\,\text{m s}^{-1}$. The driver continues at her speed for 2 s after passing S. She then reduces her speed to $22\,\text{m s}^{-1}$ with constant deceleration of $3\,\text{m s}^{-2}$, and continues at the lower speed.

a Draw a speed–time graph to illustrate the motion of the car after it passes S.

b Find the shortest distance before the road works that S should be placed on the road to ensure that a car driven in this way has had its speed reduced to $22\,\text{m s}^{-1}$ by the time it reaches the start of the road works.

20 A train starts from rest at station A and accelerates uniformly at $3x\,\text{m s}^{-2}$ until it reaches a speed of $30\,\text{m s}^{-1}$. For the next T seconds the train maintains this constant speed. The train then retards uniformly at $x\,\text{m s}^{-2}$ until it comes to rest at a station B. The distance between the stations is 6 km and the time taken from A to B is 5 minutes.

a Sketch a speed–time graph to illustrate this journey.

b Show that $\frac{40}{x} + T = 300$.

c Find the value of T and the value of x.

d Calculate the distance the train travels at constant speed.

e Calculate the time taken from leaving A until reaching the point half-way between the stations.

Summary of key points.

1 You need to know these symbols and what they represent.

s	displacement (distance)
u	starting (initial) velocity
v	final velocity
a	acceleration
t	time

2 If a particle is slowing down it has a negative acceleration. This is called deceleration or retardation.

3 Convert all your measurements into base SI units before substituting values into the formulae.

Measurement	**SI unit**
time (t)	seconds (s)
displacement (s)	metres (m)
velocity (v or u)	metres per second (m s^{-1})
acceleration (a)	metres per second per second (m s^{-2})

4 You need to remember the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\frac{u + v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

5 An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\,\text{m s}^{-2}$.

6 The gradient of a speed–time graph illustrating the motion of a particle represents the acceleration of the particle.

7 The area under a speed–time graph illustrating the motion of a particle represents the distance moved by the particle.

8 Area of a trapezium = average of the parallel sides × height

$$= \frac{1}{2}(a + b) \times h$$

9 At constant speed, distance = speed × time

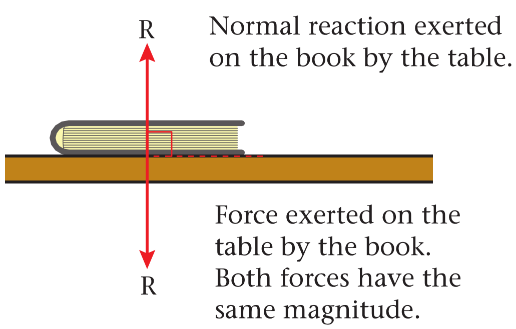
After completing this chapter you should be able to

- solve problems involving the forces acting on a body moving in a straight line with constant velocity or constant acceleration
- resolve a force into its components
- understand the coefficient of friction and solve problems involving friction
- solve problems involving collisions between bodies
- understand the Impulse – Momentum principle.

3

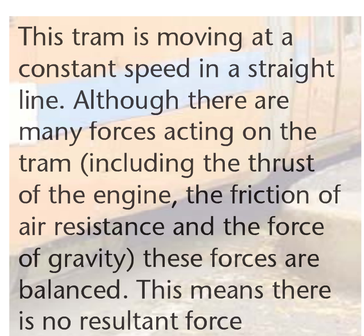
Dynamics of a particle moving in a straight line

A force is what causes an object or particle to accelerate. Newton's first law of motion states that an object will remain at rest or will continue to move in a straight line at a constant speed unless it is acted upon by a resultant force.



Newton's third law of motion states that every action has an equal and opposite reaction. This means that if a body *P* exerts a force on a body *Q*, then *Q* exerts an equal force on *P* in the opposite direction.

This book exerts a downward force of magnitude R on the table. The table is also exerting an upward force of equal magnitude on the book. This is called the normal reaction between the book and the table.


This tram is moving at a constant speed in a straight line. Although there are many forces acting on the tram (including the thrust of the engine, the friction of air resistance and the force of gravity) these forces are balanced. This means there is no resultant force

3.1 You can use Newton's Laws and the formula $F = ma$ to solve problems involving force and acceleration.

■ **Newton's second law of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced.**

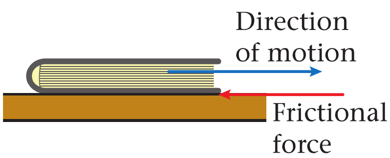
The unit of force is the **newton** (**N**). It is defined as the force that will cause a mass of 1 kg to accelerate at a rate of $1\,\text{m s}^{-2}$.

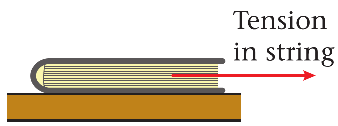
■ $\boldsymbol{F = ma}$

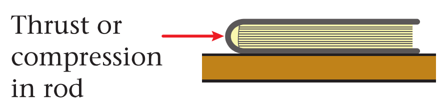
This **equation of motion** gives a relationship between the resultant force, $\boldsymbol{F}$ newtons, acting on a particle of mass $\boldsymbol{m}$ kg producing an acceleration of $\boldsymbol{a}$ m s^{-2}.

You need to be able to understand the different types of force that can act on an object.

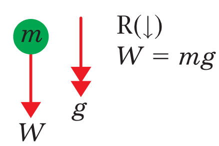
- The **normal reaction** is the force which acts perpendicular to a surface when an object is in contact with the surface. In the example on the previous page, the normal reaction is equal in magnitude to the weight of the book.
- **Friction** is a force which opposes the motion between two rough surfaces.
- If an object is being pulled along by a string, the force acting on the object is called the **tension** in the string.
- If an object is being pushed along using a light rod, the force acting on the object is called the **thrust** or **compression** in the rod.







- An object moving through air or fluid will experience **resistance** due to friction between the object and the air or fluid. This force opposes the motion of the object.
- **Gravity** is the force between any object and the Earth. The force due to gravity acting on an object is called the **weight** of the object, and it acts vertically downwards. A body falling freely experiences an acceleration of $g = 9.8\,\text{m s}^{-2}$. Using the relationship $\boldsymbol{F} = \boldsymbol{ma}$ we can write a formula for the weight of a body of mass $\boldsymbol{m}$.



■ $\boldsymbol{W = mg}$

When there is more than one force acting on an object you can **resolve** the forces in a certain direction to find the resultant force in that direction. **You usually resolve in the direction of the acceleration and perpendicular to the direction of the acceleration.**

In your answers, you should use the letter R, together with an arrow, e.g. R(↑) to indicate the direction in which you are resolving the forces.

Example 1

Find the weight in newtons of a particle of mass 12 kg.

$W = mg$

$= 12 \times 9.8$

$= 117.6$

The particle weighs 120 N, correct to two significant figures.

Your value of g is correct to two significant figures. Give your answer to the same degree of accuracy.

Example 2

Find the acceleration when a particle of mass 1.5 kg is acted on by a resultant force of 6 N.

$F = ma$

$6 = 1.5a$

$a = 4$

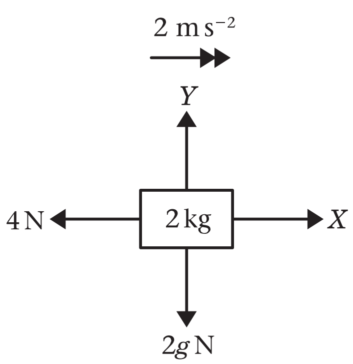
The acceleration is $4\ \text{m s}^{-2}$.

Substitute the values you know and solve the equation to find a.

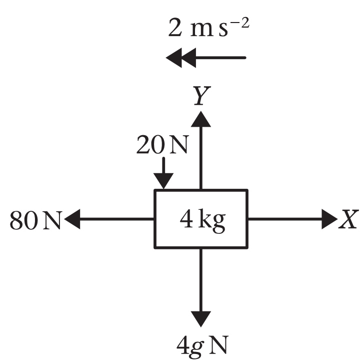
Example 3

In each of these diagrams the body is accelerating as shown. Find the magnitudes of the unknown forces X and Y.

a



b



a $R(\rightarrow)$, $X - 4 = 2 \times 2$

$X = 8$ N

R(→), means apply $F = ma$ in the direction of the arrow i.e. the arrow is the positive direction.

On one side of the equation we include all the forces, giving each one the correct sign.

On the other side, we have mass × acceleration, giving the acceleration the correct sign.

$R(\uparrow)$, $Y - 2g = 2 \times 0$

$Y = 2g$

$= 19.6$ N

There is no vertical acceleration, so $a = 0$.

b $R(\leftarrow)$, $80 - X = 4 \times 2$

$X = 72$ N

It is usually easier to take the positive direction as the direction of the acceleration.

$R(\uparrow)$, $Y - 20 - 4g = 4 \times 0$

$Y = 20 + (4 \times 9.8)$

$= 59.2$ N

$= 59$ N (2 s.f.)

There is no vertical acceleration, so $a = 0$.

Exercise 3A

Remember that g should be taken as 9.8 m s^{-2}.

1 Find the weight in newtons of a particle of mass 4 kg.

2 Find the mass of a particle whose weight is 490 N.

3 The weight of an astronaut on the Earth is 686 N. The acceleration due to gravity on the Moon is approximately 1.6 m s^{-2}. Find the weight of the astronaut when he is on the Moon.

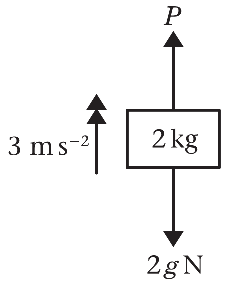
4 Find the force required to accelerate a 1.2 kg mass at a rate of 3.5 m s^{-2}.

5 Find the acceleration when a particle of mass 400 kg is acted on by a resultant force of 120 N.

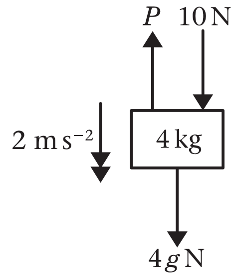
6 An object moving on a rough surface experiences a constant frictional force of 30 N which decelerates it at a rate of 1.2 m s^{-2}. Find the mass of the object.

7 In each of the following scenarios, the forces acting on the body cause it to accelerate as shown. Find the magnitude of the unknown forces P and Q.

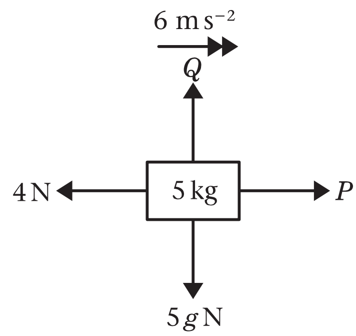
a



b

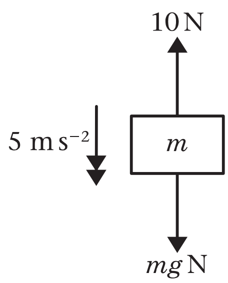


c



8 In each of the following situations, the forces acting on the body cause it to accelerate as shown. In each case find the mass of the body, m.

a



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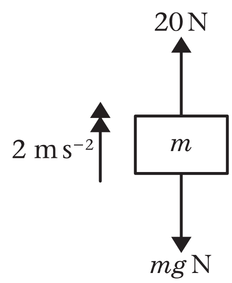
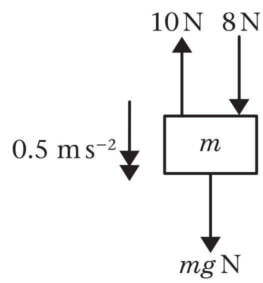
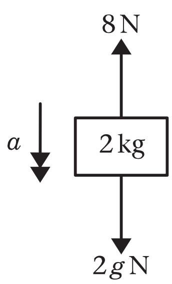


c

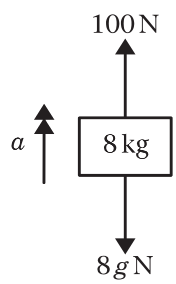


9 In each of the following situations, the forces acting on the body cause it to accelerate as shown with magnitude a m s^{-2}. In each case find the value of a.

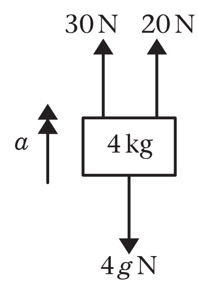
a



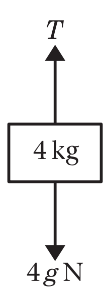
b



c



10 The diagram shows a block of mass 4 kg attached to a vertical rope.



Find the tension in the rope when the block moves downwards **a** with an acceleration of 2 m s^{-2}, **b** at a constant speed of 4 m s^{-1}, **c** with a deceleration of 0.5 m s^{-2}.

3.2 You can solve problems involving forces by drawing a diagram showing all the relevant forces and resolving in one or more directions as necessary.

Example 4

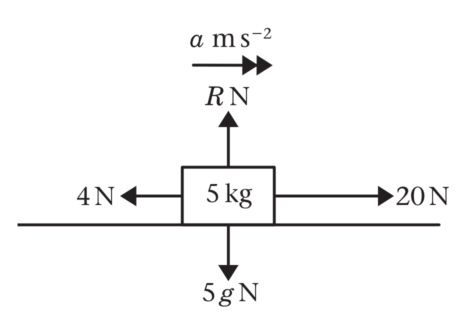
A particle of mass 5 kg is pulled along a rough horizontal table by a horizontal force of magnitude 20 N against a constant friction force of magnitude 4 N. Given that the particle is initially at rest find

a the acceleration of the particle,

b the distance travelled by the particle in the first 4 seconds,

c the magnitude of the normal reaction between the particle and the table.

a



Draw a diagram showing all the forces and the acceleration.

$R(\rightarrow)$, $20 - 4 = 5a$

Resolving horizontally, taking our positive direction as the direction of the acceleration, and using $F = ma$.

$$a = \frac{16}{5} = 3.2$$

The particle accelerates at 3.2 m s^{-2}.

b $s = ut + \frac{1}{2}at^2$

Since the acceleration is constant.

$s = (0 \times 4) + \frac{1}{2} \times 3.2 \times 4^2$

Substituting in the values.

$= 25.6$

The particle moves a distance of 25.6 m.

c $R(\uparrow)$, $R - 5g = 5 \times 0 = 0$

Resolving vertically, taking up as positive. Write all the forces on the left-hand-side of the equation and mass × acceleration on the right-hand-side.

$R = 5g = 5 \times 9.8 = 49$ N

The normal reaction has magnitude 49 N.

Example 5

A small pebble of mass 500 g is attached to the lower end of a light string. Find the tension in the string when the pebble

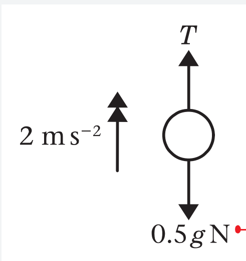
a is moving upwards with an acceleration of $2\,\text{m s}^{-2}$,

b is moving downwards with an acceleration of $3\,\text{m s}^{-2}$,

c is moving downwards at a constant speed of $5\,\text{m s}^{-1}$.

d is moving downwards with a deceleration of $4\,\text{m s}^{-2}$.

a



Draw a diagram showing all the forces and the acceleration.

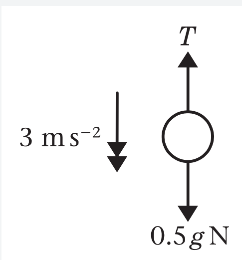
Convert 500 g into 0.5 kg first.

$R(\uparrow)$, $T - 0.5g = 0.5 \times 2$

$T = 5.9\,\text{N}$

Resolving vertically upwards, in the direction of the acceleration.

b



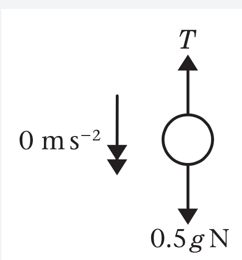
Draw a diagram showing the forces and acceleration.

$R(\downarrow)$, $0.5g - T = 0.5 \times 3$

$T = 3.4\,\text{N}$

Resolving vertically downwards, in the direction of the acceleration.

c

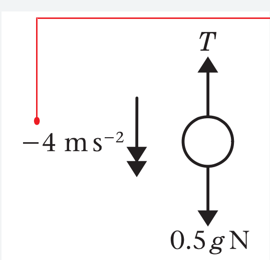


$R(\downarrow)$, $0.5g - T = 0.5 \times 0$

$T = 4.9\,\text{N}$

Constant speed means zero acceleration.

d



Deceleration is shown as negative acceleration.

$R(\downarrow)$, $0.5g - T = 0.5 \times -4$

$T = 6.9\,\text{N}$

Resolving, taking downwards as positive.

Exercise 3B

1 A ball of mass 200 g is attached to the upper end of a vertical light rod. Find the thrust in the rod when it raises the ball vertically with an acceleration of $1.5\,\text{m s}^{-2}$.

2 A small pebble of mass 50 g is dropped into a pond and falls vertically through it with an acceleration of $2.8\,\text{m s}^{-2}$. Assuming that the water produces a constant resistance, find its magnitude.

3 A lift of mass 500 kg is lowered or raised by means of a metal cable attached to its top. The lift contains passengers whose total mass is 300 kg. The lift starts from rest and accelerates at a constant rate, reaching a speed of $3\,\text{m s}^{-1}$ after moving a distance of 5 m. Find

a the acceleration of the lift,

b the tension in the cable if the lift is moving vertically downwards,

c the tension in the cable if the lift is moving vertically upwards.

4 A block of mass 1.5 kg falls vertically from rest and hits the ground 16.6 m below after falling for 2 s. Assuming that the air resistance experienced by the block as it falls is constant, find its magnitude.

5 A trolley of mass 50 kg is pulled from rest in a straight line along a horizontal path by means of a horizontal rope attached to its front end. The trolley accelerates at a constant rate and after 2 s its speed is $1\,\text{m s}^{-1}$. As it moves, the trolley experiences a resistance to motion of magnitude 20 N. Find

a the acceleration of the trolley,

b the tension in the rope.

6 A trailer of mass 200 kg is attached to a car by a light tow-bar. The trailer is moving along a straight horizontal road and decelerates at a constant rate from a speed of $15\,\text{m s}^{-1}$ to a speed of $5\,\text{m s}^{-1}$ in a distance of 25 m. Assuming there is no resistance to the motion, find

a the deceleration of the trailer,

b the thrust in the tow-bar.

7 A woman of mass 60 kg is in a lift which is accelerating upwards at a rate of $2\,\text{m s}^{-2}$.

a Find the magnitude of the normal reaction of the floor of the lift on the woman.

The lift then moves at a constant speed and then finally decelerates to rest at $1.5\,\text{m s}^{-2}$.

b Find the magnitude of the normal reaction of the floor of the lift on the woman during the period of deceleration.

c Hence explain why the woman will feel heavier during the period of acceleration and lighter during the period of deceleration.

8 The engine of a van of mass 400 kg cuts out when it is moving along a straight horizontal road with speed $16\,\text{m s}^{-1}$. The van comes to rest without the brakes being applied.

In a model of the situation it is assumed that the van is subject to a resistive force which has constant magnitude of 200 N.

a Find how long it takes the van to stop.

b Find how far the van travels before it stops.

c Comment on the suitability of the modelling assumption.

9 Albert and Bella are both standing in a lift. The mass of the lift is 250 kg. As the lift moves upward with constant acceleration, the floor of the lift exerts forces of magnitude 678 N and 452 N respectively on Albert and Bella. The tension in the cable which is pulling the lift upwards is 3955 N.

a Find the acceleration of the lift.

b Find the mass of Albert.

c Find the mass of Bella.

10 A small stone of mass 400 g is projected vertically upwards from the bottom of a pond full of water with speed $10\,\text{m s}^{-1}$. As the stone moves through the water it experiences a constant resistance of magnitude 3 N. Assuming that the stone does not reach the surface of the pond, find

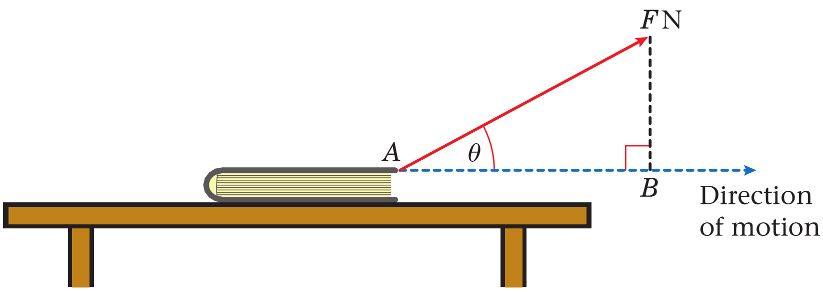
a the greatest height above the bottom of the pond that the stone reaches,

b the speed of the stone as it hits the bottom of the pond on its return,

c the total time taken for the stone to return to its initial position on the bottom of the pond.

3.3 If a force is applied at an angle to the direction of motion you can resolve it to find the component of the force that acts in the direction of motion.

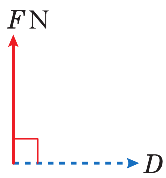
This book is being dragged along the table by means of a force of magnitude F. The book is moving horizontally, and the angle between the force and the direction of motion is θ.



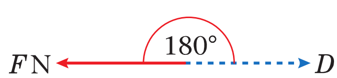
The effect of the force in the direction of motion is the length of the line AB. This is called the **component of the force in the direction of motion**. Using the rule for a right-angled triangle $\cos\theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$ you can see that the length of AB is $F \times \cos\theta$. Finding this value is called **resolving** the force in the direction of motion.

- **The component of a force of magnitude F in a certain direction is $F\cos\theta$, where θ is the size of the angle between the force and the direction.**

If F acts in the direction D, then the component of F in that direction is $F\cos 0° = F \times 1 = F$



If F acts at the right angles to D, then the component of F in that direction is $F\cos 90° = F \times 0 = 0$

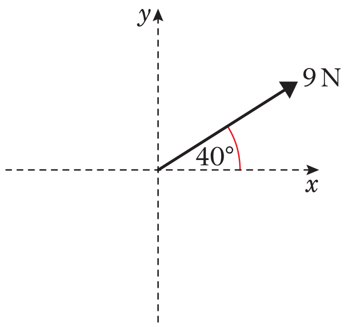


If F acts in the opposite direction to D, then the component of F in that direction is $F\cos 180° = F \times -1 = -F$

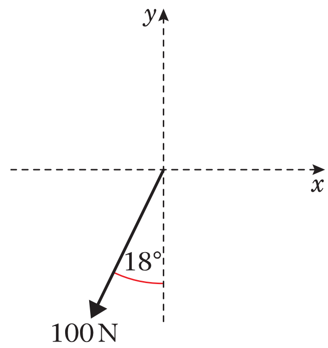
Example 6

Find the component of each force in **i** the x-direction, **ii** the y-direction.

a



b



a **i** $\theta = 40°$

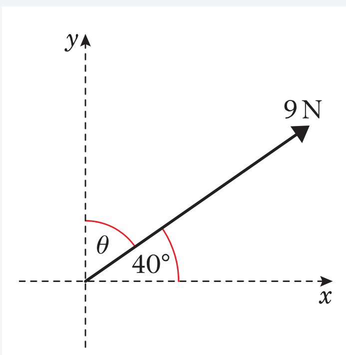
Component in x-direction $= F \cos \theta$

$= 9 \times \cos 40°$

$= 6.89$ N (3 s.f.)

Give your answers correct to three significant figures.

ii



$\theta = 90° - 40°$

$= 50°$

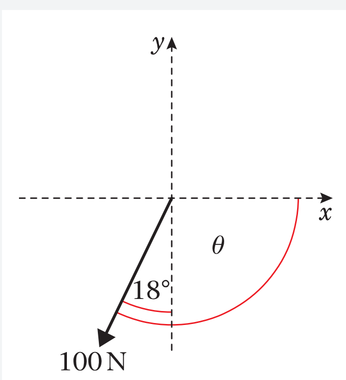
Make sure you find the angle between the force and the direction you are resolving in.

Component in y-direction $= F \cos \theta$

$= 9 \times \cos 50°$

$= 5.79$ N (3 s.f.)

b **i**



$\theta = 90° + 18°$

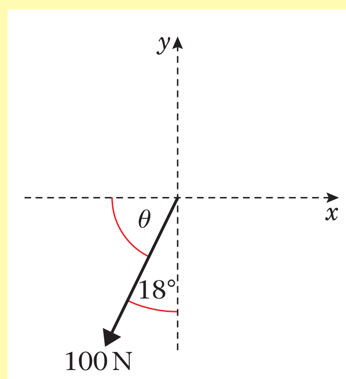
$= 108°$

Component in x-direction $= F \cos \theta$

$= 100 \times \cos 108°$

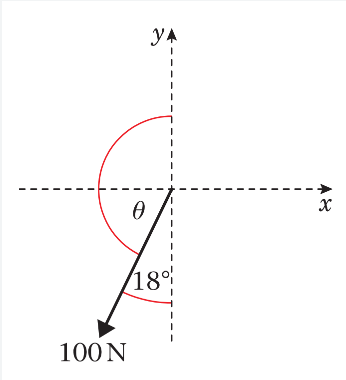
$= -30.9$ N (3 s.f.)

You are resolving in the positive x-direction so your answer is going to be negative.



You could resolve in the negative x-direction using $\theta = 90° - 18°$, then change the sign of your answer from positive to negative, i.e. component $= -100 \cos 72°$ $= -30.9$ N (3 s.f.)

ii



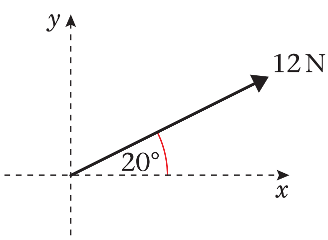
$\theta = 180° - 18°$
$= 162°$

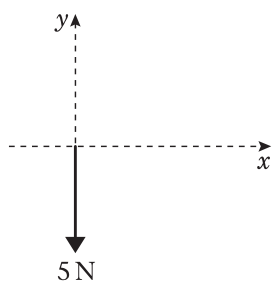
You could use $\theta = 18°$ then change the sign of your answer from positive to negative, i.e. component $= -100 \cos 18°$ $= -95.1$ N (3 s.f.).

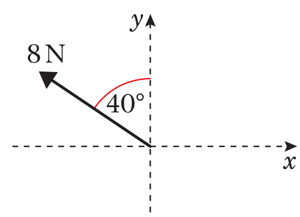
Component in y-direction $= F \cos \theta$
$= 100 \times \cos 162°$
$= -95.1$ N (3 s.f.)

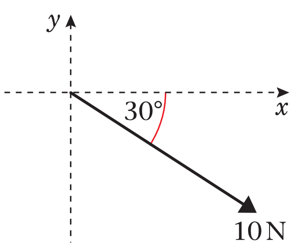
Exercise 3C

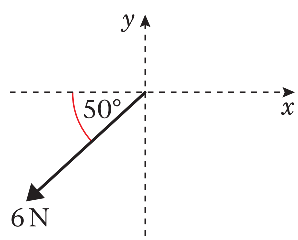
1 Find the component of each force in
i the x-direction, **ii** the y-direction.

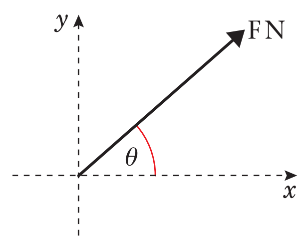
a


b


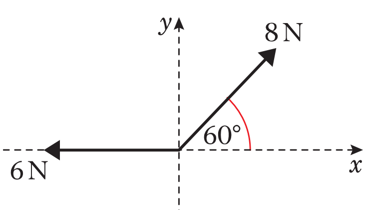
c


d


e


f


2 For each of the following systems of forces, find the sum of the components in
i the x-direction, **ii** the y-direction.

a


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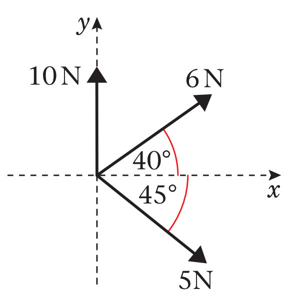
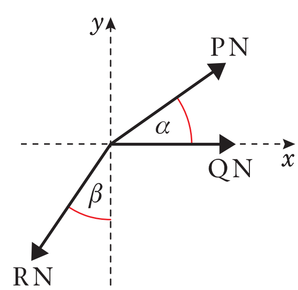
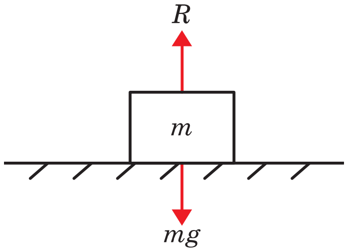


c


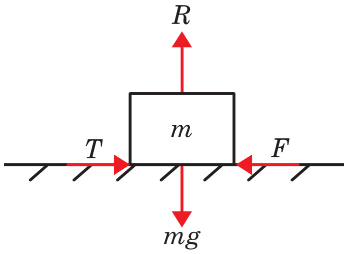
3.4 You can calculate the magnitude of a frictional force using the coefficient of friction.

Friction is a force which opposes motion between two rough surfaces. It occurs when the two surfaces are moving relative to one another, or when there is a **tendency** for them to move relative to one another.

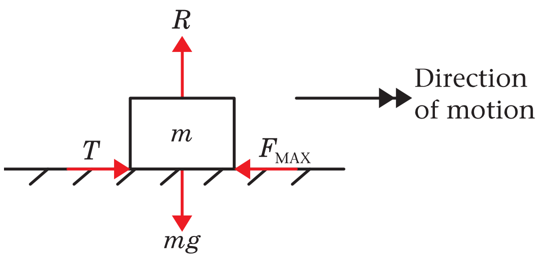
This block is stationary. There is no horizontal force being applied, so there is no tendency for the block to move. There is no frictional force acting on the block.



This block is also stationary. There is a horizontal force being applied which is not sufficient to move the block. There is a tendency for the block to move, but it doesn't because the force of friction is equal and opposite to the force being applied.



As the applied force increases, the force of friction increases to prevent the block from moving. If the magnitude of the applied force exceeds a certain **maximum** or **limiting value**, the block will move. While the block moves, the force of friction will remain constant at its maximum value.



The limiting value of the friction depends on two things:

- the normal reaction R between the two surfaces in contact,
- the roughness of the two surfaces in contact.

You can measure roughness using the **coefficient of friction**, which is represented by the letter μ (pronounced *myoo*). The rougher the two surfaces are, the larger the value of μ. For smooth surfaces there is no friction and $\mu = 0$.

■ **The maximum or limiting value of the friction F_{MAX} between two surfaces is given by**

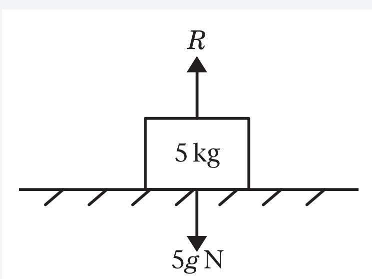
$$F_{MAX} = \mu R$$

where μ is the coefficient of friction and R is the normal reaction between the two surfaces.

Example 7

Find the maximum frictional force which can act on a block of mass 4 kg which rests on a rough horizontal plane, if the coefficient of friction between the block and the plane is **a** 0.2, **b** 0.7.

a



First draw a diagram showing all the forces acting on the block.

$R(\uparrow)$, $R - 4g = 4 \times 0 = 0$

$R = 39.2$ N

$F_{MAX} = \mu R = 0.2 \times 39.2$

$= 7.84 = 7.8$ N (2 s.f.)

There is no acceleration upwards.

You can now calculate the maximum 'available' friction force.

b As before, $R = 39.2$ N

$F_{MAX} = \mu R = 0.7 \times 39.2$

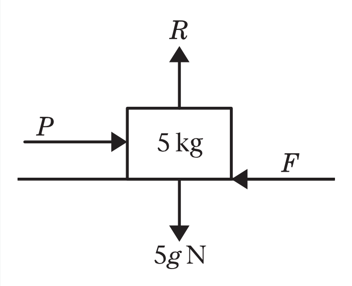
$= 27.44 = 27$ N (2 s.f.)

You now have a different value for μ.

Since μ is higher there is more friction 'available'.

Example 8

A block of mass 5 kg lies at rest on rough horizontal ground. The coefficient of friction between the block and the ground is 0.4. A horizontal force P is applied to the block. Find the magnitude of the friction force acting on the block and the acceleration of the block when the magnitude of P is
a 10 N, **b** 19.6 N, **c** 30 N.



$R(\uparrow)$, $R = 5g = 49$ N

So $F_{MAX} = \mu R = 0.4 \times 49$

$= 19.6$ N

The maximum available friction force is 19.6 N.

First draw a diagram showing all the forces acting on the block.

You then need to calculate what the maximum possible friction force is in this situation.

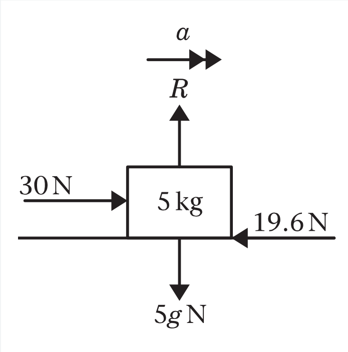
The normal reaction will equal the weight as there is no acceleration upwards.

Do not round this value as you will need to use it in your calculations.

a When $P = 10$ N, the friction will only need to be 10 N to prevent the block from sliding and the block will remain at rest in equilibrium.

b When $P = 19.6$ N, the friction will need to be at its maximum value of 19.6 N to prevent the block from sliding, and the block will remain at rest in **limiting equilibrium**.

c When $P = 30$ N, the friction will be unable to prevent the block from sliding, but it will remain at its maximum value of 19.6 N. The block will accelerate from rest along the plane in the direction of P with acceleration a.



$R(\rightarrow)$, $30 - 19.6 = 5a$

$a = 2.08 = 2.1\,\text{m s}^{-2}$ (2 s.f.)

Draw a diagram showing all the forces and the acceleration.

Resolving in the direction of the acceleration.

The answer is rounded to two significant figures as we have used $g = 9.8\,\text{m s}^{-2}$.

- **If a force *P* is applied to a block of mass *m* which is at rest on a rough horizontal surface and *P* acts at an angle to the horizontal:**
  - **the normal reaction *R* is not equal to *mg*,**
  - **the force tending to pull or push the block along the plane is not equal to *P*.**

Example 9

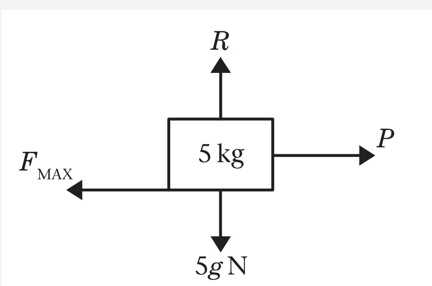
A 5 kg box lies at rest on a rough horizontal floor. The coefficient of friction between the box and the floor is 0.5. A force *P* is applied to the box to pull or push it horizontally along the floor. Find the magnitude of *P* which is necessary to achieve this if

a *P* is applied horizontally,

b *P* is applied at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$,

c *P* is applied at an angle α below the horizontal, where $\tan\alpha = \frac{3}{4}$.

a



$R(\uparrow)$, $R - 5g = 5 \times 0 = 0$

$R = 5g = 49\,\text{N}$

$\therefore \quad F_{MAX} = \mu R = 0.5 \times 49$

$= 24.5\,\text{N}$

In the position of limiting equilibrium there is no acceleration, so $P = F_{MAX} = 24.5\,\text{N}$

P must exceed 24.5 N.

Draw a diagram showing the forces acting.

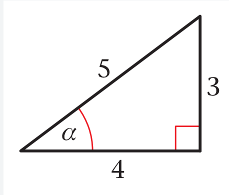
There is no acceleration upwards.

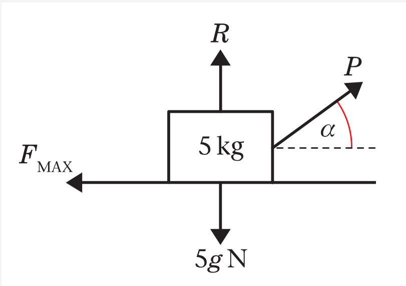
Find the maximum possible friction force.

b Note that if $\tan \alpha = \frac{3}{4}$,

then $\sin \alpha = \frac{3}{5} = 0.6$

and $\cos \alpha = \frac{4}{5} = 0.8$.





Draw a new diagram.

$R(\uparrow)$, $R + P\cos(90° - \alpha) - 5g = 0$

Again there is no acceleration upwards.

$R = 49 - P\sin\alpha = 49 - 0.6P$

$\cos(90° - \alpha) = \sin\alpha = 0.6$

$R(\rightarrow)$, $P\cos\alpha - F_{MAX} = 0$

No acceleration horizontally.

$P\cos\alpha = F_{MAX}$

$F_{MAX} = \mu R = 0.5(49 - 0.6P)$

Use the expression for R found above.

So $0.8P = 0.5(49 - 0.6P)$

Use $\cos\alpha = 0.8$.

$0.8P = 24.5 - 0.3P$

Multiply out.

$1.1P = 24.5$

Collect the P terms.

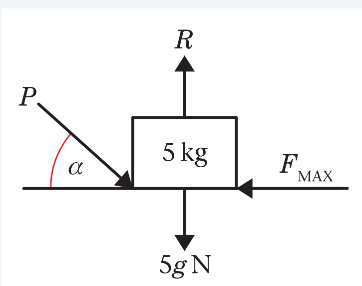
$P = 22\text{ N}$ (2 s.f.)

Solve for P.

P must exceed 22 N.

Note that this is less than the value found in part **a**.

c



Draw another diagram.

$R(\uparrow)$, $R - 5g - P\cos(90° - \alpha) = 5 \times 0$
$= 0$

There is no acceleration vertically.

$R = 5g + P\sin\alpha$
$= 49 + 0.6P$

Use $\cos(90° - \alpha) = \sin\alpha$ as before.

$F_{MAX} = \mu R = 0.5(49 + 0.6P)$
$= 24.5 + 0.3P$

Use the expression for R found above.

$R(\rightarrow)$, $P\cos\alpha - F_{MAX} = 5 \times 0 = 0$

No acceleration horizontally.

$0.8P = 24.5 + 0.3P$

Substituting for F_{MAX} from above.

$0.5P = 24.5$

$P = 49\text{ N}$

P must exceed 49 N.

Note the size of this answer compared with **b**.

Exercise 3D

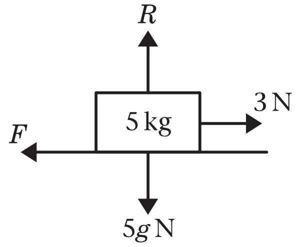
1 Each of the following diagrams shows a body of mass 5 kg lying initially at rest on rough horizontal ground. The coefficient of friction between the body and the ground is $\frac{1}{7}$. In each diagram R is the normal reaction from the ground on the body and F is the friction force exerted on the body by the ground. Any other forces applied to the body are as shown on the diagram. In each case

i find the magnitude of F,

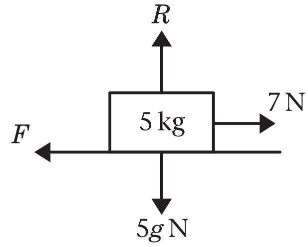
ii state whether the body will remain at rest or accelerate from rest along the ground,

iii find, when appropriate, the magnitude of this acceleration.

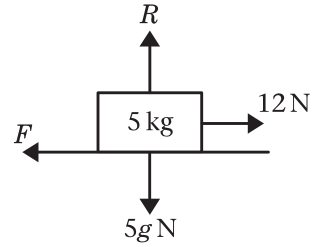
a



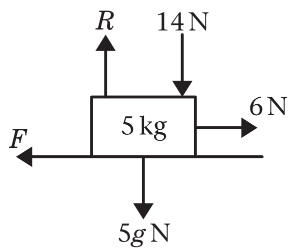
b



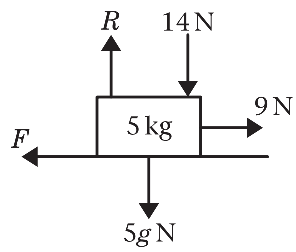
c



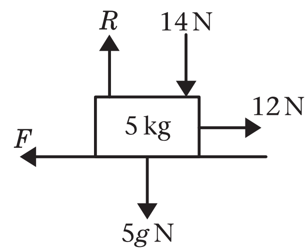
d



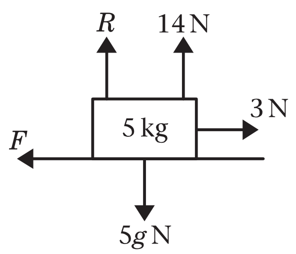
e



f



g



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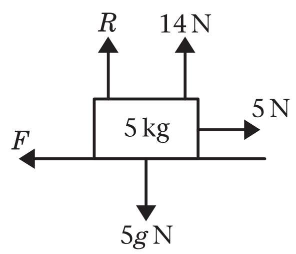
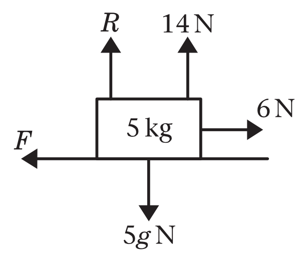


i



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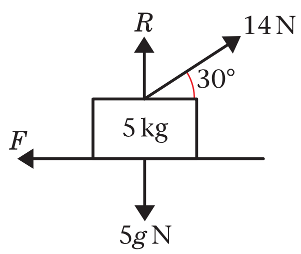

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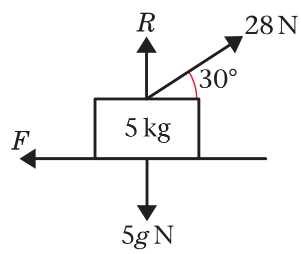

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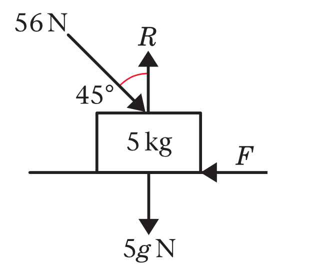
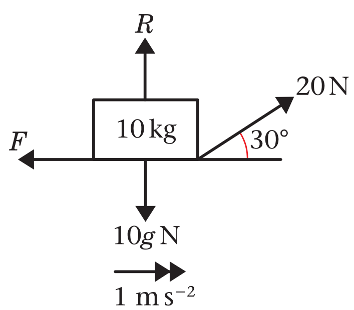
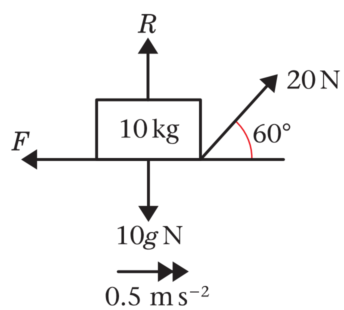


2 In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. R is the normal reaction and F is the friction force. Find the coefficient of friction in each case.

a



b



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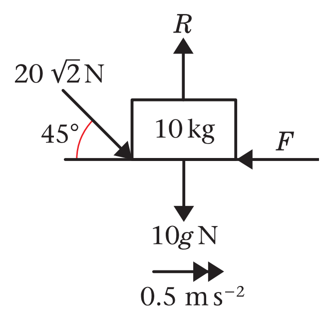
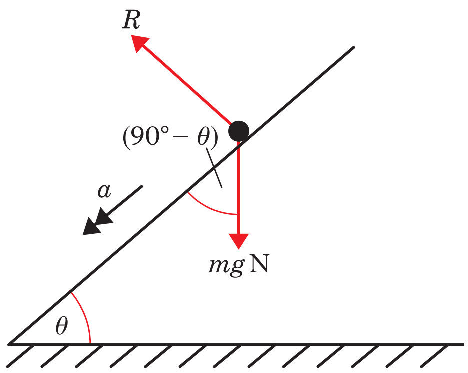


3.5 You can solve problems about a particle on an inclined plane by resolving the forces parallel and perpendicular to the plane.

■ **If a particle of mass m is placed on a smooth inclined plane and released it will slide down the slope.**

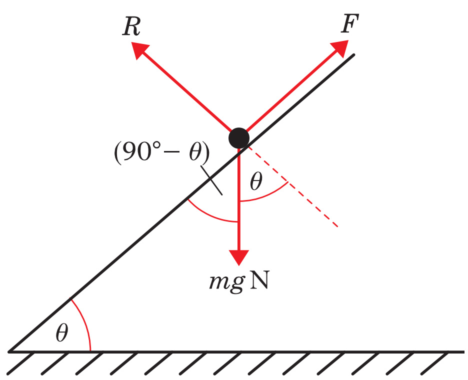


Resolve in the direction of motion, which is down the slope.

$$R(\swarrow) \quad \cancel{m}g\cos(90° - \theta) = \cancel{m}a$$
$$g\cos(90° - \theta) = a$$
$$g\sin\theta = a$$

The mass of the particle does not affect the acceleration, but the angle of the slope does.

If the plane is rough, the force of friction might be sufficient to prevent the particle from moving.



Resolve perpendicular to the plane. Remember that the normal reaction acts at right-angles to the plane.

$$R(\nwarrow) \qquad R - mg\cos\theta = m \times 0 = 0$$
$$R = mg\cos\theta$$

Now resolve in the direction of the slope. If the particle is stationary:

$$R(\nearrow) \quad mg\cos(90° - \theta) - F = m \times 0 = 0$$
$$mg\sin\theta = F$$

The frictional force F is always less than or equal to F_{MAX}.

$$F \leqslant \mu R$$
$$\cancel{mg}\sin\theta \leqslant \mu\cancel{mg}\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} \leqslant \mu$$
$$\tan\theta \leqslant \mu$$

■ **A particle placed on a rough inclined plane will remain at rest if tan $\theta \leqslant \mu$, where θ is the angle the plane makes with the horizontal and μ is the coefficient of friction between the particle and the plane.**

If $\tan\theta > \mu$ then the particle will accelerate down the slope.

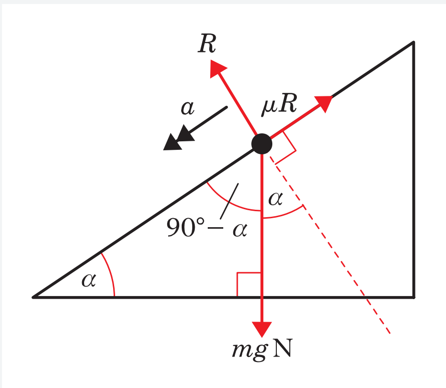
Example 10

A particle is held at rest on a rough plane which is inclined to the horizontal at an angle α, where $\tan \alpha = 0.75$. The coefficient of friction between the particle and the plane is 0.5. The particle is released and slides down the plane. Find

a the acceleration of the particle,

b the distance it slides in the first 2 seconds.

a



Draw a diagram showing all the forces and the acceleration. Note that you are not given the mass of the particle so call it m.

Since the particle slides down the plane friction will be limiting, so $\mu R = 0.5R$

$R(\nwarrow)$, $R - mg\cos\alpha = m \times 0 = 0$

$R = mg\cos\alpha$ ①

Resolving perpendicular to the acceleration.

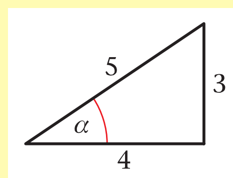
$R(\swarrow)$, $mg\sin\alpha - \mu R = ma$ ②

Resolving in the direction of the acceleration.

From equation ①,

$R = 0.8mg$

If $\tan\alpha = 0.75 = \frac{3}{4}$,



$\cos\alpha = \frac{4}{5} = 0.8$

and $\sin\alpha = \frac{3}{5} = 0.6$

(or use your calculator)

Then equation ② becomes

$0.6\cancel{m}g - 0.5 \times (0.8\cancel{m}g) = \cancel{m}a$

$0.6g - 0.4g = a$

$0.2g = a$

Substitute for $\sin\alpha$, μ and R and cancel the m's.

The acceleration of the particle is $0.2g$ or $1.96\ \text{m s}^{-2}$ (3 s.f.) or $2.0\ \text{m s}^{-2}$ (2 s.f.)

Any of these answers would be acceptable.

b Since the acceleration is *constant* we can use a formula:

$u = 0,\ a = 0.2g,\ t = 2,\ s = ?$

$s = ut + \frac{1}{2}at^2$

Choose the appropriate formula.

$s = 0 + \frac{1}{2} \times 0.2g \times 2^2$

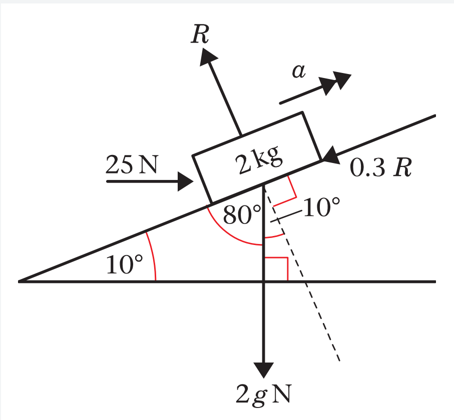
$= 3.92 = 3.9$ (2 s.f.)

Substitute in the values.

The particle slides 3.9 m (2 s.f.) down the plane.

Example 11

A box of mass 2 kg is pushed up a rough plane by a horizontal force of magnitude 25 N. The plane is inclined to the horizontal at an angle of 10°. Given that the coefficient of friction between the box and the plane is 0.3, find the acceleration of the box.



Draw a diagram showing all forces and the acceleration. Note that friction will be limiting, since the box is sliding.

$R(\nwarrow)$, $R - 2g\cos 10° - 25\cos 80° = 2 \times 0 = 0$

$R = 2g\cos 10° + 25\cos 80°$ ①

Resolving perpendicular to the acceleration.

$R(\nearrow)$, $25\cos 10° - 2g\cos 80° - 0.3R = 2a$

Resolving in the direction of the acceleration.

$25\cos 10° - 2g\cos 80° - 0.3(2g\cos 10° + 25\cos 80°) = 2a$

Substituting for R from equation ① and simplifying.

$(25 - 0.6g)\cos 10° - (2g + 7.5)\cos 80° = 2a$

$14.124\ldots = 2a$

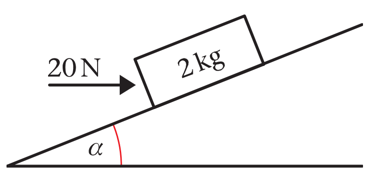
$a = 7.1\text{ m s}^{-2}$ (2 s.f.)

The box accelerates up the plane at 7.1 m s^{-2} (2 s.f.).

Exercise 3E

1 A particle of mass 0.5 kg is placed on a smooth inclined plane. Given that the plane makes an angle of 20° with the horizontal, find the acceleration of the particle.

2 The diagram shows a box of mass 2 kg being pushed up a smooth plane by a horizontal force of magnitude 20 N. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.



Find

a the normal reaction between the box and the plane,

b the acceleration of the box up the plane.

3 A boy of mass 40 kg slides from rest down a straight slide of length 5 m. The slide is inclined to the horizontal at an angle of 20°. The coefficient of friction between the boy and the slide is 0.1. By modelling the boy as a particle, find

a the acceleration of the boy,

b the speed of the boy at the bottom of the slide.

4 A block of mass 20 kg is released from rest at the top of a rough slope. The slope is inclined to the horizontal at an angle of 30°. After 6 s the speed of the block is $21\ \text{m s}^{-1}$. As the block slides down the slope it is subject to a constant resistance of magnitude R N. Find the value of R.

5 A book of mass 2 kg slides down a rough plane inclined at 20° to the horizontal. The acceleration of the book is $1.5\ \text{m s}^{-2}$. Find the coefficient of friction between the book and the plane.

6 A block of mass 4 kg is pulled up a rough slope, inclined at 25° to the horizontal, by means of a rope. The rope lies along the line of the slope. The tension in the rope is 30 N. Given that the acceleration of the block is $2\ \text{m s}^{-2}$ find the coefficient of friction between the block and the plane.

7 A parcel of mass 10 kg is released from rest on a rough plane which is inclined at 25° to the horizontal.

a Find the normal reaction between the parcel and the plane.

Two seconds after being released the parcel has moved 4 m down the plane.

b Find the coefficient of friction between the parcel and the plane.

8 A particle P is projected up a rough plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{3}$. The particle is projected from the point A with speed $20\ \text{m s}^{-1}$ and comes to instantaneous rest at the point B.

a Show that while P is moving up the plane its deceleration is $\dfrac{13g}{15}$.

b Find, to three significant figures, the distance AB.

c Find, to three significant figures, the time taken for P to move from A to B.

d Find the speed of P when it returns to A.

3.6 You can solve problems involving connected particles by considering the particles separately.

If a system involves the motion of more than one particle, the particles may be considered separately. Particular care is then needed to ensure that all the forces acting on each particle are considered.

Provided that all parts of the system are moving in the **same straight line**, then you can also treat the whole system as a single particle.

Example 12

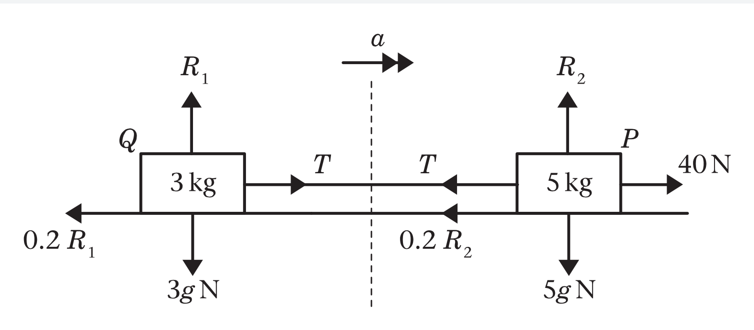
Two particles P and Q, of masses 5 kg and 3 kg respectively, are connected by a light inextensible strong. Particle P is pulled by a horizontal force of magnitude 40 N along a rough horizontal plane. The coefficient of friction between each particle and the plane is 0.2. The string is taut.

a Find the acceleration of each particle.

b Find the tension in the string.

c Explain how the modelling assumptions that the string is light and inextensible have been used.

a



Each mass will have a different normal reaction and therefore a different friction.

For Q: R(↑), $R_1 - 3g = 0$

$R_1 = 3g$ N

For P: R(↑), $R_2 - 5g = 0$

$R_2 = 5g$ N

No acceleration upwards.

For the whole system (P and Q):

You can do this because both masses are moving in the same straight line.

R(→), $40 - (0.2 \times 3g) - (0.2 \times 5g) = 8a$

$40 - 1.6g = 8a$

$a = 3.04\text{ m s}^{-2}$

The tension in the string is not included as you are treating the whole system as a single particle.

The acceleration of each mass is 3.04 m s^{-2} since the string is inextensible.

b For Q: R(→), $T - 0.2 \times 3g = 3 \times 3.04$

$T = 15$ N

To find T consider one of the particles. It is easier to consider Q rather than P.

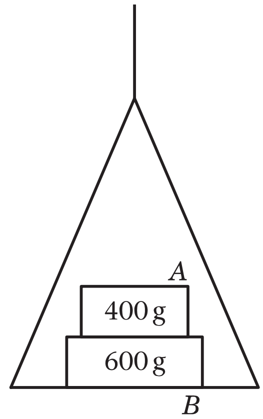
The tension in the string is 15 N.

c Inextensible ⇒ acceleration of masses is the same.

light ⇒ tension is the same throughout the length of the string and the mass of the string is negligible.

Example 13

A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400 g and the mass of B is 600 g. A rests on top of B, as shown in the diagram.



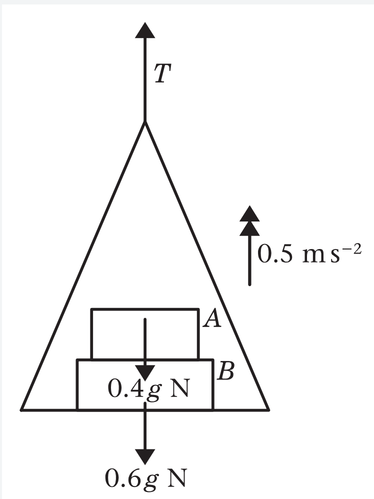
The scale-pan is raised vertically, using the string, with acceleration $0.5\ \text{m s}^{-2}$.

a Find the tension in the string.

b Find the force exerted on mass B by mass A.

c Find the force exerted on mass B by the scale-pan.

a



For the whole system:

You can use this since all parts of the system are moving in the same straight line.

$R(\uparrow)\quad T - 0.4g - 0.6g = (0.4 + 0.6)a$

Note that we must convert 400 g to 0.4 kg and 600 g to 0.6 kg.

so, $T - g = 1 \times 0.5$

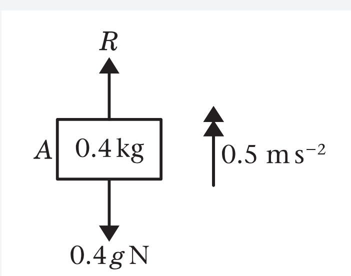
$a = 0.5$.

$T = 10.3\ \text{N}$

Simplify.

The tension in the string is 10.3 N (3 s.f.) or 10 N (2 s.f.).

b



For A only:

$R(\uparrow)\quad R - 0.4g = 0.4 \times 0.5$

We find the force exerted on A by B and then use Newton's 3rd Law to say that the force exerted on B by A will have the same magnitude.

$R = 4.12\ \text{N}$

So the force exerted on B by A is 4.12 N (3 s.f.) or 4.1 N (2 s.f.).

3.7 You can calculate the momentum of a particle and the impulse of a force.

- **The momentum of a body of mass m which is moving with velocity v is mv.**

If m is in kg and v is in m s^{-1} then the momentum will be kg m s^{-1}. However, since $\text{kg m s}^{-1} = (\text{kg m s}^{-2})\,\text{s}$ and kg m s^{-2} are the units for force ($F = ma$) you can also measure momentum in N s.

Velocity is a vector quantity and mass is a scalar, so momentum is a vector quantity.

Example 17

Find the magnitude of the momentum of

a a cricket ball of mass 400 g moving at $18\,\text{m s}^{-1}$,

b a lorry of mass 5 tonnes moving at $0.3\,\text{m s}^{-1}$.

a Momentum = mass × velocity

Magnitude of momentum $= \frac{400}{1000} \times 18 = 7.2\,\text{N s}$

The mass must be in kg.

The units can be N s or kg m s^{-1}.

b Momentum = mass × velocity

Magnitude of momentum $= (5 \times 1000) \times 0.3 = 1500\,\text{kg m s}^{-1}$

The mass must be in kg.

The units can be N s or kg m s^{-1}.

■ **If a constant force F acts for time t then we define the impulse of the force to be Ft.**

If F is in N and t is in s then the units of impulse will be N s.

Force is a vector quantity and time is a scalar, so impulse is a vector quantity.

Examples of an impulse include a bat hitting a ball, a snooker ball hitting another ball or a jerk in a string when it suddenly goes tight. In all these cases the time for which the force acts is very small but the force is quite large and so the product of the two, which gives the impulse, is of reasonable size. However, there is no theoretical limit on the size of t.

Suppose a body of mass m is moving with an initial velocity u and is then acted upon by a force F for time t. This results in its final velocity being v.

Its acceleration is given by $a = \dfrac{v - u}{t}$.

Substituting into $F = ma$:

$$F = m\left(\frac{v-u}{t}\right)$$

$$Ft = m(v - u)$$

$$= mv - mu$$

The impulse of the force I is given by: $I = Ft$.

■ $$I = mv - mu$$

Impulse = Final momentum − Initial momentum

Impulse = Change in momentum

This is a vector equation, so a positive direction must be chosen and each value given the correct sign.

This is called the **Impulse–Momentum Principle**.

Example 18

A body of mass 2 kg is initially at rest on a smooth horizontal plane. A horizontal force of magnitude 4.5 N acts on the body for 6 s. Find

a the magnitude of the impulse given to the body by the force,

b the final speed of the body.

a Magnitude of the impulse = force × time

$= 4.5 \times 6 = 27\ \text{N s}$.

The units can be N s or kg m s^{-1}.

b Impulse = Final momentum − Initial momentum

$27 = 2v - 0$

The body is at rest initially.

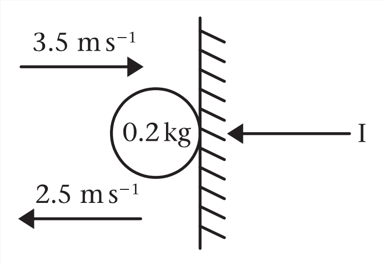
$v = 13.5\ \text{m s}^{-1}$

Example 19

A ball of mass 0.2 kg hits a fixed vertical wall at right angles with speed 3.5 m s^{-1}. The ball rebounds with speed 2.5 m s^{-1}. Find the magnitude of the impulse exerted on the wall by the ball.

Because the wall is fixed you cannot apply the Impulse–Momentum Principle to it. Find the magnitude of the impulse exerted on the ball by the wall and then use Newton's 3rd Law to deduce that the magnitude of the impulse exerted on the wall by the ball with be the same.

This diagram shows the initial and final velocities of the ball and the impulse acting on it.



Note that this is a plan view of the situation.

Choose a positive direction and apply the Impulse–Momentum Principle to the ball.

$(\leftarrow)\quad I = (0.2 \times 2.5) - (0.2 \times (-3.5))$

The initial velocity is in the negative direction.

$= 0.5 + 0.7$

$= 1.2\ \text{N s}$

Therefore, by Newton's 3rd Law, the magnitude of the impulse exerted on the wall by the ball is 1.2 N s.